Application nb.1

SURVEYING CALCULATIONS

Delivery date Student

24.02.2009

Application nb.1

Surveying calculations

The ensemble of operations in order to obtain plans and maps is known as surneying.Because during land surveying, the calculations involve a large volume of work, it is proper that the way of doing these calculations to be well studied.

Application theme

In order to establish the position of points, depending on a reference coordinates system, related to the Earth's surface, it is necessary the knowledge of some essential surveying calculations.

Datums

1. The space rectangular coordinates of the 32 and 43 points from the triangulation network.
2. The results of measurements for establishing the space position of a new point, 107: the horizontal angle, β, the zenith angle, Z, the ground distance, d_i, which is measured directly using a 50 m long steel measuring tape.

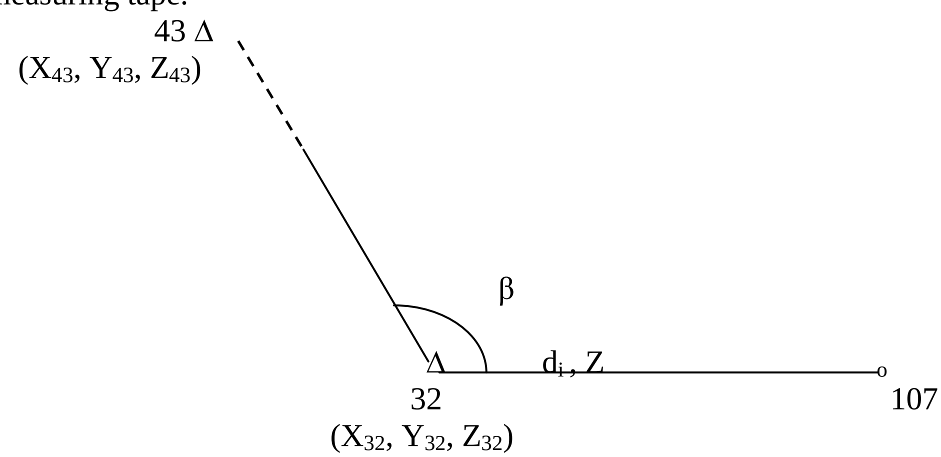


Fig. 1.1

The applications datums containg the initial elements i. e. , space rectangular coordinates of the 32 and 43 points, and the elements resulting from field measurements as the horizontal and zenith angles and the ground distance are indicated in table.

NOTE: The datums of the application will be modified as below:

- the abscisa of the 32 point: X_{32} +/-n(m)
- the ground distance: d_i +/-n(m)

Table 1.1

Pct. statie	Pct. vizat	Distanta Inclinata d_i	Unghi Zenital Z	Unghi Orizontal β	Coordonate spatiale			Nr. pct.
					X	Y	Z	
45	51	-	-	-	2421,69	1833,12	231,96	43
	101	138.75	94^g51^c	131^g74^c	1735,43	2351,12	164,55	32

The application will consist of :

1. Determination of the grid bearing and horizontal distance of 32 – 43 its coordinates .
2. Determination , by transfer, of the 32 – 107 direction's grid bearing .
3. Determination of the distance reduced to the horizontal d_o and the level difference between the known point , 32 and the new one , 107 .
4. Determination of the relative plane rectangular coordinates x and y of the 107 point .
5. Determination of the space rectangular coordinates of the 107 point .

Solution

The topographic plan is an orthogonal projection . The points position on the plan is established in a plane rectangular coordinates system . For Romania , according to the stereographic projection 1970 , the general plane rectangular coordinates system was obtained considering the abscisa X as the plane projection of the central point's meridian situated north of Fagaras (N_{go}) , and the ordinate Y perpendicularly on the abscisa in the central point (fig . 1. 2) . This way the coordinates axis have the same orientation as the cardinal points (the X axis on S – N direction and Y axis on W – E direction.) .
In the conformal traverse cylindrical projection (Gauss' s projection) , the X axis is represented by the plane projection of the central meridian , and the image of terrestrial equator in the form of a straight line at right angle to the axial (central) meridian serves as Y axis.
In the zonal system, the origin of coordinates for all points in the given zone is assumed to the intersection of the central meridian with the equator . (fig . 1. 3).
In surveying the direction according to which all the directions in field are determinated is represented by the north (N) direction .
On the Earth's surface , through every point we have a certain geographical (or true) meridian that has a stable position and a certain magnetic meridian which is not stable in time .
As a reference direction we will take a parallel to the geographical meridian of the central point (in the case of stereographic projection) and to the central meridian (in the case of Gauss's projection) . This way , the grid bearing of a direction of line A-B , written θ_{AB} , is used to obtain the direction of line A-B with reference to the central meridian , or to a line parallel to it .
The grid bearing is reckoned clockwise from the central meridian or from a line parallel to it up to the direction , of a given line within 0^g to 400^g (or 0^o to 350^o) .

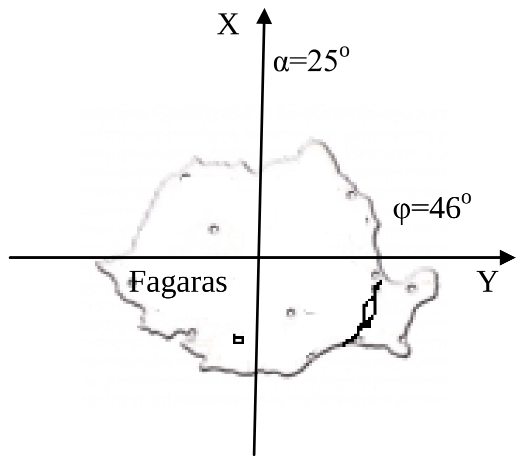


Fig . 1.2

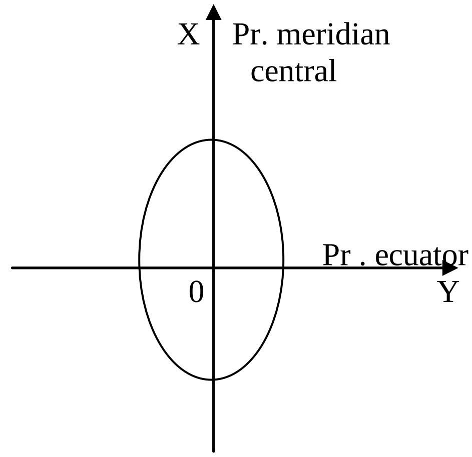


Fig . 1.3

In figure 1.4. it is shown the AB line grid bearing . We have the following notations :
A – the geographical azimuth with reference to the geographical meridian of point A .
A_m - the magnetic azimuth with reference to the geographical meridian of point A .
Θ_{AB} – grid bearing

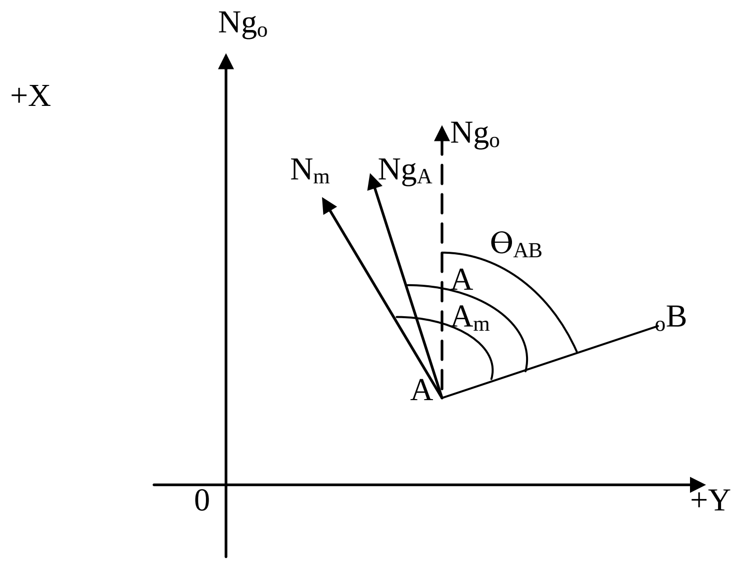
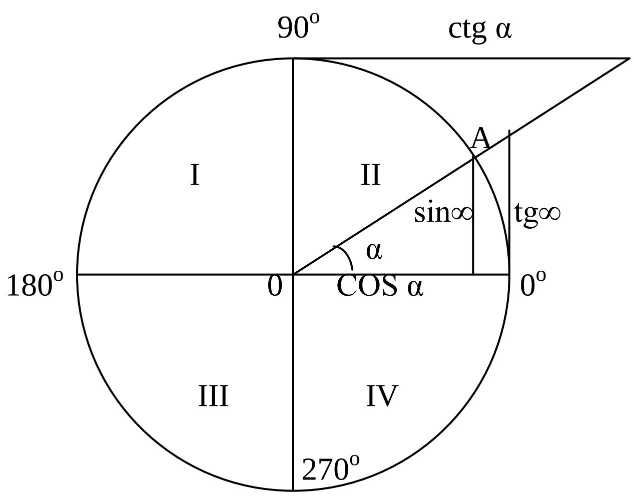
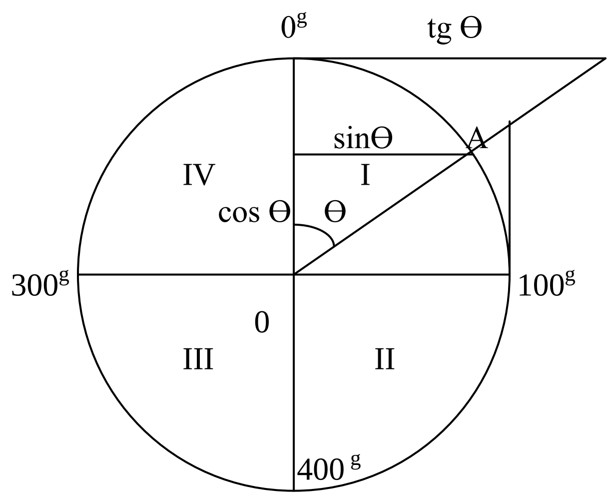


Fig . 1. 4.

Since the position of the points is established trigonometrycly , it was necessary the replacemen of the trigonometryc circle with the topographic one .



TRIGONOMETRIC CIRCLE



TOPOGRAPHIC CIRCLE

At this circle the origin of measuring the grid bearings is the N – direction and the numbering of quadrants and reckoning of angles is pursued clockwise , i. e . , rightward .

The grid bearing of a certain direction night be in different quadrants . An important role has in this case the calculation angle , β_i (i=I, II, III, IV) , actually a reduced angle at the first quadrant . It is always an acute angle between the given direction and the nearest extremity of the geographical meridian (of the X axis) . The index of the calculation angle indicates the quadrant in which the grid bearing is situated (fig . 1.7) .

Table 1. 2

Quadr.	Grid bearings variation intervals	Conections between grid bearings and calculation angles	Name of the calculation angle
I	$O^g<=\theta_A<=100^g$	$\Theta_A-B_I=O^g$	NE
II	$100^g<=\theta_B<=200^g$	$\Theta_B+B_{II}=200^g$	SE
III	$200^g<=\theta_C<=300^g$	$\Theta_C-B_{III}=200^g$	SV
IV	$300^g<=\theta_D<=400^g$	$\Theta_D+B_{IV}=400^g$	NV

For the convergence from the grid bearing to the calculation angle and conversely we can use table 1 . 2 and fig . 1. 7 .

A fundamental principle is surveying is horizontal ground distances reduction using the zenith angle measured with a radio direction finder . When the ground distance d_i has been measured with a steel measuring tape crea measuring reel , the horizontal reduction is made with one of the formulas given below :

$d_O=d_i*\sin z$ or $d_O=di*\cos x$ (Fig. 18

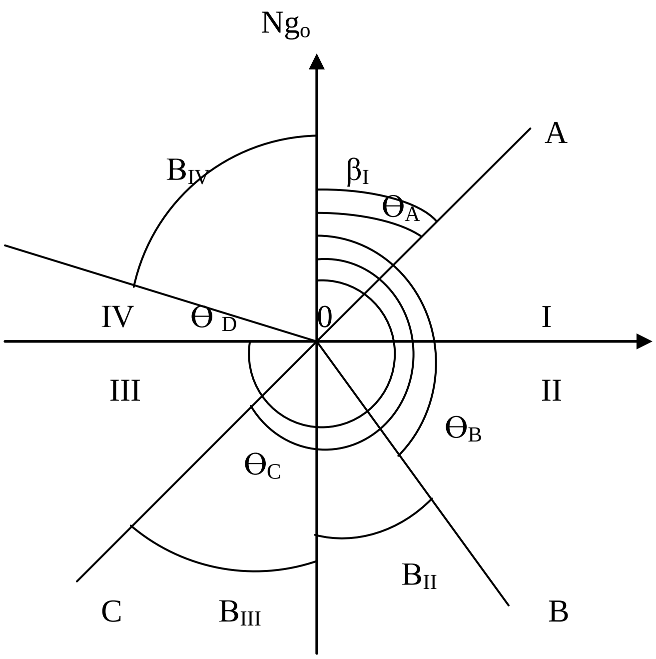


Fig . 17

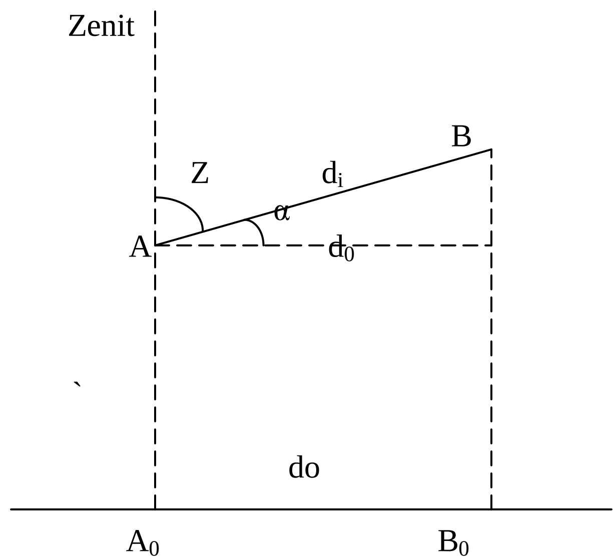


Fig . 18

Using the polar coordinates , θ_{AB} grid bearing and the distance reduced to the horizontal . d_o, the relative coordinates of a new point B are calculated with reference to a previous point A using the formulas (Fig. 19) .

$\Delta x_{AB}=d_o*\cos\theta_{AB}$
$\Delta y_{AB}=d_o*\sin\theta_{AB}$

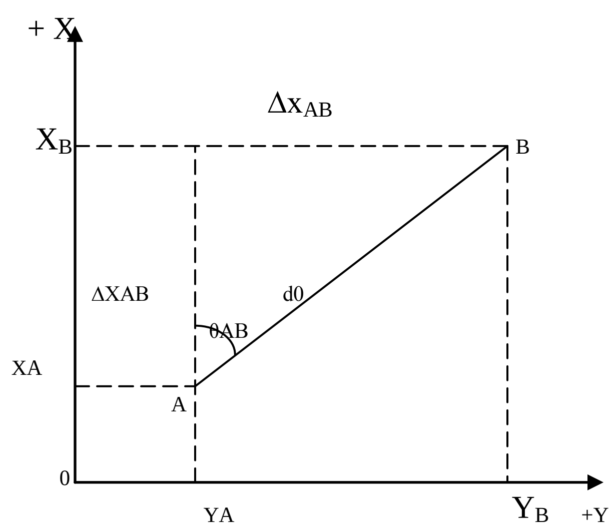


FIG 19

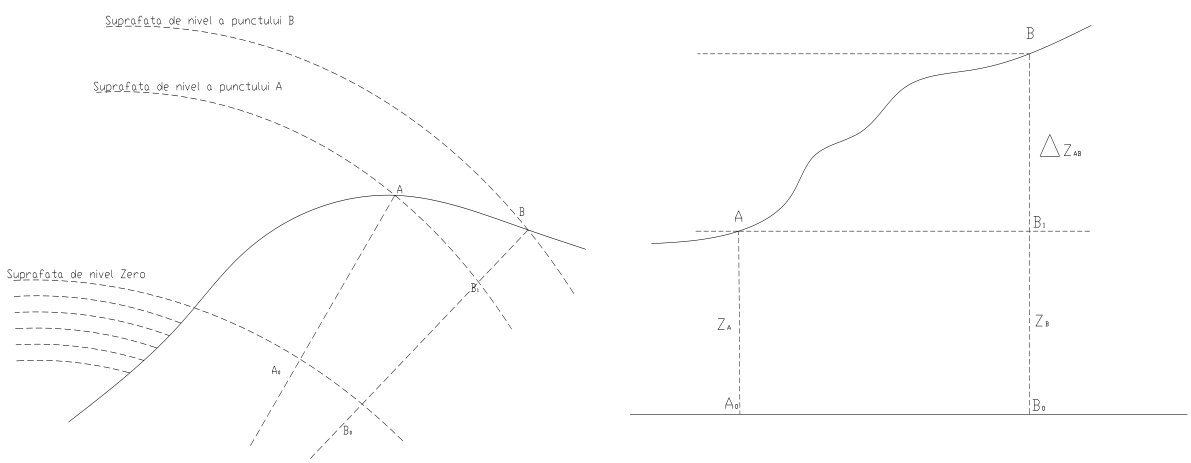
The relative coordinates signs are given by the cosinus and sinus trigonometrical functions, these functions being related to the quadrant in wich the grid bearing is situated . The plane rectangular coordinates (absolute coordinates) of the new point , B , will be :

$$X_B=X_A+\Delta_{XAB}$$
$$Y_B=Y_A+\Delta_{YAB}$$

For the establishing of the point's in space position it is necessary the determination of its elevations , i. e. , its vertical position . This is determinated being given a reference surface , represented by the mean sea level (geoid's surface) which is called altitude datum level surface . So , all the elevations of all points are established with reference to this mark .

For Romania , the chart datum mark is situated in the Constantza harbour, the level system being called Black Sea level system .

In fig. 1. 10. a. it is presented the points elevations determinations in the case of large areas when the level surfaces are approximately spheres with the same center , and in fig. 1. 10. b. the case of small areas when the level surfaces can be regarded as parallel horizontal planes .



a) b)

Fig . 1. 10

The absolute elevation or the altitude of point is the height between the altitude datum level surface and the level surface of the considered point ($Z_A=AA_0$; $Z_B=BB_0$) .

The difference of level between two points is the height between the level surfacea belonging to these two points ($\Delta Z_{AB}=BB_1$) .

The difference of level between points , which is also a relative coordinates, is calculated using the field measurement, with the following espressions :

$\Delta z_{AB}=d_i*\cos Z=d_0*\text{ctg } Z$

or

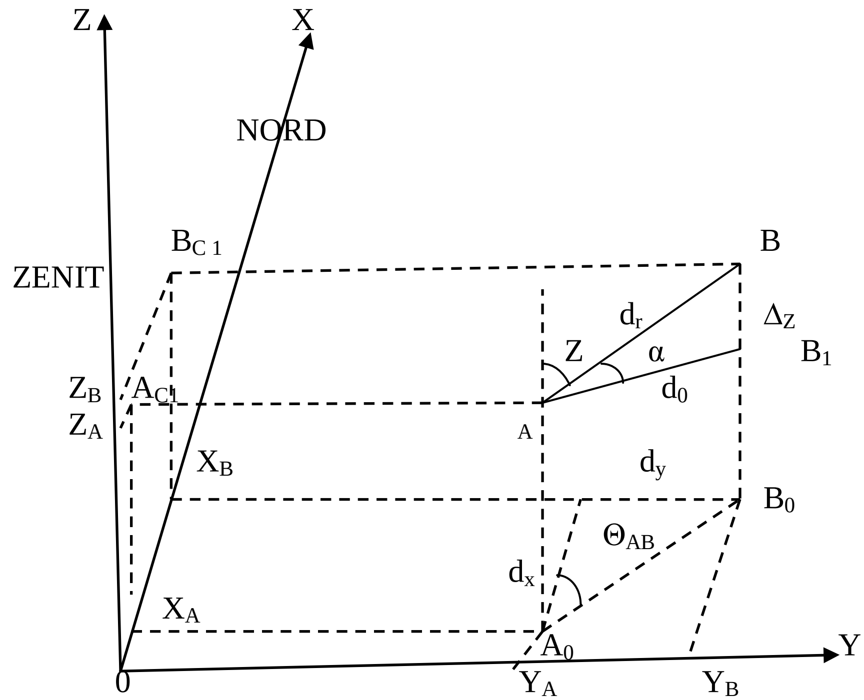
$\Delta z_{AB}=d_i*\sin Z=d_0*\text{tg } \infty$

The sign of the level difference being given by the sign of the cosinus functions ($Z \in [\, 0^g, 200^g]$) or by the sign of the slope angle which is positive when the aim is above horizont and negative when the aim is under the horizont line .
The value of the absolute coordinate of the new point , B , with reference to the known absolute coordinate of the previous point A will be :

$$Z_B=Z_A+\Delta z_{AB}$$

According to what we have shown , a new point's position is determinated with reference to the known point , A, using space rectangular coordinates (fig. 1.11) :

$$X_B=X_A+\Delta x_{AB}$$
$$Y_B=Y_A+\Delta y_{AB}$$
$$Z_B=Z_A+\Delta z_{AB}$$



Where ,
$\Delta x_{AB}=d_0 * \cos \theta_{AB}$
$\Delta y_{AB}=d_0 * \sin \theta_{AB}$
$\Delta z_{AB}=d_0 * \mathrm{ctg}\ Z$

Application's solving will be conducted this way :

1. Dteterminations of the grid bearing and horizontal distance of 32 – 43 by its coordinates .
Being given 32 and 43 points by its plane rectangular coordinates (table 1.1, column 6 and 7) first well calculate the relative coordinates :
$\Delta x_{32\text{-}43}=X_{43}-X_{32}=3420.15-2617.91=+802.24$ m
$\Delta y_{32\text{-}43}=Y_{43}-Y_{32}=2673.42-3241.57=-568.15$ m
Because $\Delta_{X>0}$ and $\Delta_{Y<0}$, according to fig. 1.12. and 1.13 and table 1.3 , the grid bearing 32-43 is in the IV quadrant. The calculations angle β_{IV} is determinated using the following formula :

$$\beta_{IV}=\mathrm{arctg}\left|\frac{\Delta y_{32\text{-}43}}{\Delta x_{32\text{-}43}}\right|=\mathrm{arctg}\left|\frac{-\ 568.15}{802.24}\right|= 39^g22^c92^{cc}$$

Table 1. 3

Relative coordinates x	y	Quadr.	Calculation angle determination	Grid bearing
$+\Delta x$	$+\Delta y$	I	-when l+/-+Δxl>l+/-Δy we'll have β_i=arctg(l+/-Δ_Yl / l+/-Δxl)	$\Theta=\beta_I$
$-\Delta x$	$+\Delta y$	II		$\Theta=200^g-\beta_{II}$
$-\Delta x$	$-\Delta y$	III	-when l+/-Δxl<l+/-Δ_Yl we'll have β_i = arctg (l+/-Δx l / l+/-Δ_Yl)	$\Theta=200^g+\beta_{III}$
$+\Delta x$	$-\Delta y$	IV		$\Theta=400^g-\beta_{IV}$

In the fourth quadrant the grid bearing will be :
$\Theta_{32\text{-}43}=400^g-39^g22^c92^{cc}=380^g77^c08^{cc}$
The distance reduced the horizontal is determinated as it follows :

$$d_0=\frac{X_{32\text{-}43}}{\cos\Theta_{32\text{-}43}}=\frac{802.24\text{ m}}{\cos 360^g77^c08^{cc}}=983.05\text{ m}$$

$$d_0=\frac{Y_{32\text{-}43}}{\sin\Theta_{32\text{-}43}}=\frac{-568.15\text{ m}}{\sin 360^g77^c08^{cc}}=983.05\text{ m}$$

The eguality of these values is also a calculum control of the grid bearing . When we're asked only for the horizontal distance we'll use :
$d_0=\sqrt{\Delta x^2_{32\text{-}43}+\Delta y^2_{32\text{-}43}}=\sqrt{802.24^2+(-568.15)^2}=983.05$ m

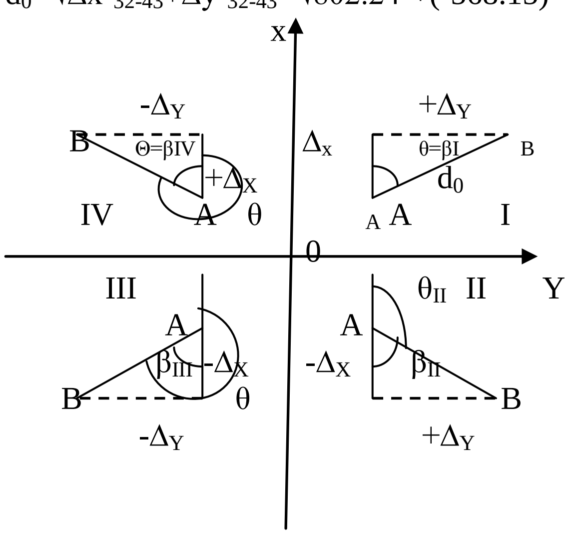


Fig. 1.12

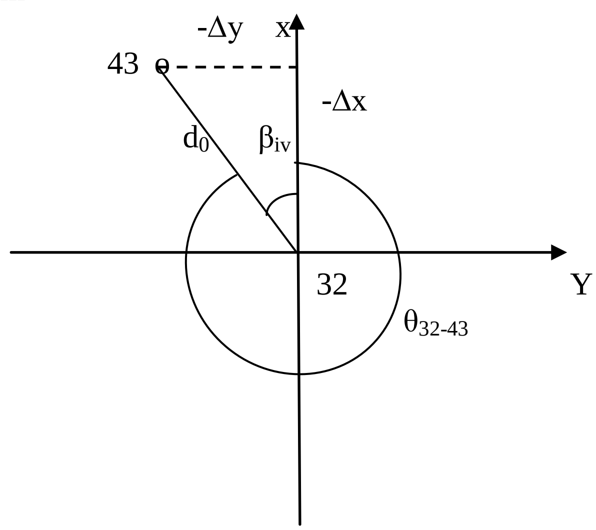


Fig. 1.13

2. Determination , by transfer, of the 32-107 direction’s grid bearing.
The grid bearing of the 32-107 direction can be determination knowing the bearing of a reference direction ($\theta_{32\text{-}43}$) and the horizontal angle , B, measured in the field by theodolite between these two directions (Table 1.1 , column 5) , using the next formula (Fig. 1.14) :

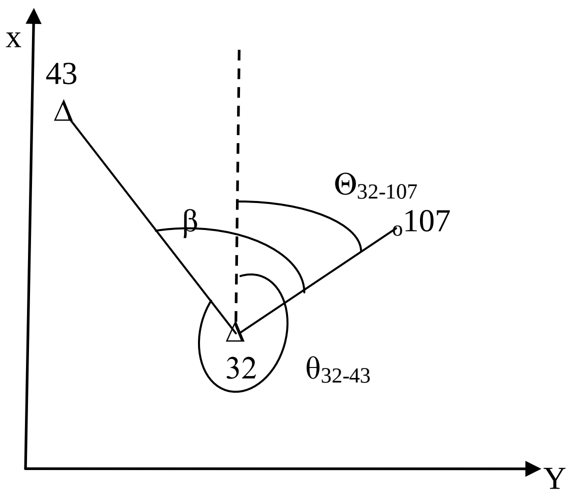


Fig. 1.14

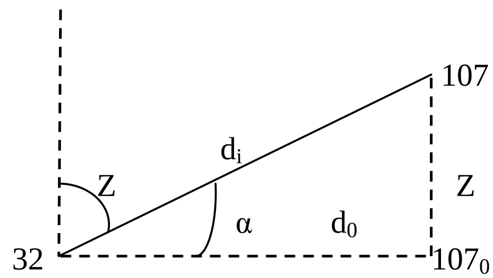
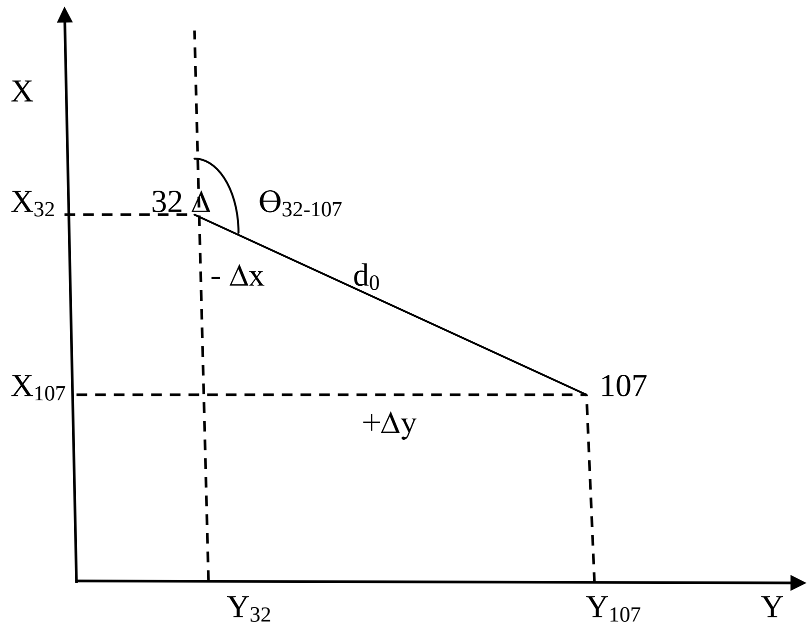


Fig. 1.15

$\theta_{32\text{-}107}=\theta_{32\text{-}43}+\beta=360^g77^c08^{cc}+147^g89^c=508^g66^c08^{cc}$
Since the ancunt outruns 400^g, as in this example, we will consider the bearing with 400^g less :
$\theta_{32\text{-}107}=508^g66^c08^{cc}-400^g=108^g66^c06^{cc}$

3. Determination of the distance reduced to the horizontal , d_0 and the level difference between the known point , 32 , and the new one , 107 .
According to the measured elements in the terrain , yhe ground distance , d_i , and the zenith angle , Z, (Table 1.1 column 3 and 4) , will calculate the distance reduced to the horizontal and the level difference using the following formulas :
$d_0=d_i*\sin Z=149.67*\sin 92^g16^c=148.54$ m
$\Delta z_{32\text{-}107}=d_i*\cos Z=149.67*\cos 92^g16^c=18.39$ m
Virtually, for level difference we’ll use :
$\Delta z_{32\text{-}107}=d_0*\text{ctg } Z=148.54*\text{ctg } 92^g16^c=18.39$ m

4. Determination of the relative plane rectangular coordinates , x and y, of the 107 point .
The relative plane rectangular coordinates are determinated using polar coordinates, the grid bearing and the distance reduced to the horizontal , using the formulas (fig. 1.16) :



$\Delta x_{32-107}=d_0*\cos\theta_{32-107}=148.54*\cos 108^g66^c08^{cc}= -20.15$ m
$\Delta y_{32-107}=d_0*\sin\theta_{32-107}=148.54*\sin 108^g66^c08^{cc}= 147.17$ m
Since the grid bearing is the third quadrant , where cosinus is negative and sinus is positive , $\Delta x<0$ and $\Delta y>0$ as it has resulted from calculation .

5. Determination of the space rectangular coordinates of the 107 point .
The space coordinates of the new point will be obtained by algebraic totalizing of the space rectangular coordinates of the knwn point 32 with the relative coordinates calculated before .
So ,
$X_{107}=x_{32}+\Delta x_{32-107}=2617.91- 20.15=2597.76$ m
$Y_{107}=y_{32}+\Delta y_{32-107}=3241.57+ 147.17=3388.74$ m
$Z_{107}=z_{32}+\Delta_{32-107}= 93.62+18.32= 112.01$ m
Finally , we can calculate , step by step , the coordinates of all details points on the topographic plane , in order to create mps and plans.

Application number 2

MAPS AND PLANS

MAPS AND PLANS

Studing and projecting of civil engineering buildings emplies using a complete topographic documentation, the most important thing being constitued of large scale and medium scale maps and plans.

During this application we will study the content of maps and plans and also the way of using them.

Application theme

For designing certain civil engineering buildings it is necessary the knowledge of the content and the ways of solutioning different problems concerning maps and plans.

Datas

1. A piece of map with 1/25000 as scale and the aquidistance e=5 m.
2. The position of two points, M and N, on the surface that has been considered.

The application will consist of:

A. Description of the piece of terrain represented on map relying on the knowledge of the conventional signs.
B. Determination of the geographic and plane rectangular coordinates of the M point & N.
C. Level determination for points M and N.
D. Determination of the horizontal distance between M and N by graphic measurement of the corresponding line on the map.
E. Determination of rate of grade (slope) of the MN line.
F. Laying out grade line on map between M and N.
G. Constructing the profile of the surface between M and N considering the length scale for 1/10000 and the elevation scale for 1/1000.

The content of maps and plans

With the aid of geometric construction it is possible to represent the contours of the ground on paper in horizontal projection thus obtaining their reduced and similar image.

The topographic map is a reduced scale representation of large portion of the earth’s surface by using conventional signs, which takes in to account the curvature of the Earth’s surface.

Maps are represented at scales smaller then 1/20000. On the map's surface the scale is not strictly constant, the variation depending on the size of the represented area and the projection system that has been considered.

The topographic plan is also a reduced scale representation of ground contours in horizontal projection with preservation of similarity and of terrain relief by using conventional signs. The curvature effect is not taken into account because of the relatively small area that is to be represented.

Plans are designed at large scale (1/200 – 1/10000) and have a great number of details exactly represented on it.

The scale is a constant ratio between the numeric value of a distance d, and the numeric value of the corresponding horizontal distance on ground D.

According to the way of presenting the scales are of two categories: numeric and graphic scales.

Numeric scale is expressed by the following formula:

$$S_c = \frac{d}{D} = \frac{1}{\frac{D}{d}} = 1:N$$

Where N is the denominator of the scale .Also, d and D have to be expressed in the same length units. For examples, 1/25000 scale means that for d=1 mm the scale we'll have a corresponding ground distance, D=25000 mm (25 m).

Scale's formula allows us to determinate an element using the other two:

$$d = \frac{D}{N}; \quad D = d \times N; \quad N = \frac{D}{d}$$

The graphic scale is a graphic representation of the numerical scale. Usually with the aid of the graphic scale we can obtain directly the size of the ground horizontal distance, D, by graphic measuring of the corresponding distance d on plan or map.

The simple (linear) graphic scale has its precision as the tenth (1/10) part of the base of the scale (fig.2.1).

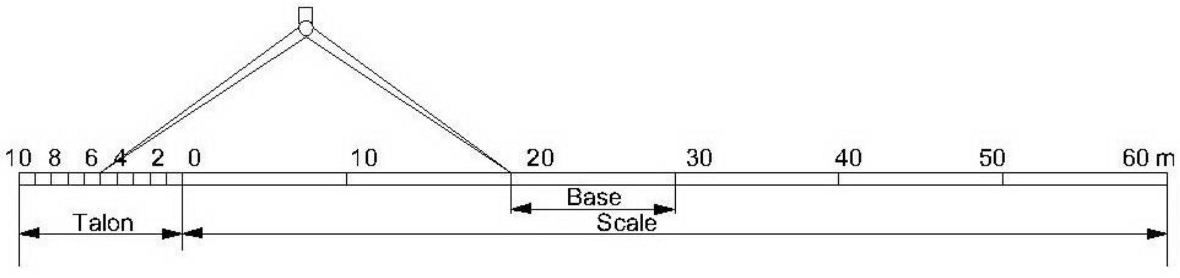


Fig.2.1

For this figure that is according to 1/10000 scale, the ground distance is D=34 m.

For raising the precision it has been made the normal surveyor's scale.

The map border is drawn on the contour of each map as filled lines, black and white, representing the grid of meridians and parallels. The map border’s lines delimit vertically 1’ latitude and horizontally 1’ longitude. In each of the four corners of the map border are written the values of longitudes and latitudes in hexadecimal degrees. Depending on the map border, we can determinate graphically on map the geographic coordinates of every point.

The kilometer grid is represented by a grid of squares, obtained by parallels at the coordinates axis spaced 1 km apart. On the border ok ma, having the directions of the two axes, we have the kilometric values of each line. Depending on the kilometer grid the point’s plane rectangular coordinates are found.

Setting up the plane is made using a small graphic in the leftmost down corner (fig.2.3) which contains three directions: the geographic north direction, the magnetic north direction and the geographic north direction of the projection center, the last one corresponding to the X-axis of the rectangular coordinates system. Between these directions we have: the convergence of the meridians. Y, between the geographic meridian of the considered point and a parallel to the geographic meridian of the projection center (N of Fagaraș) ; the magnetic declination, ɗ , between the geographic and the magnetic meridian in the considered point on map (fig.2.2). Depending on these angles we

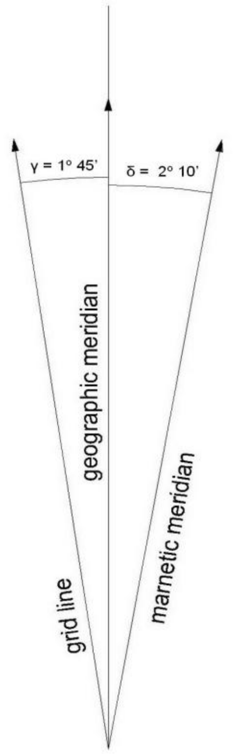


Fig.2.2

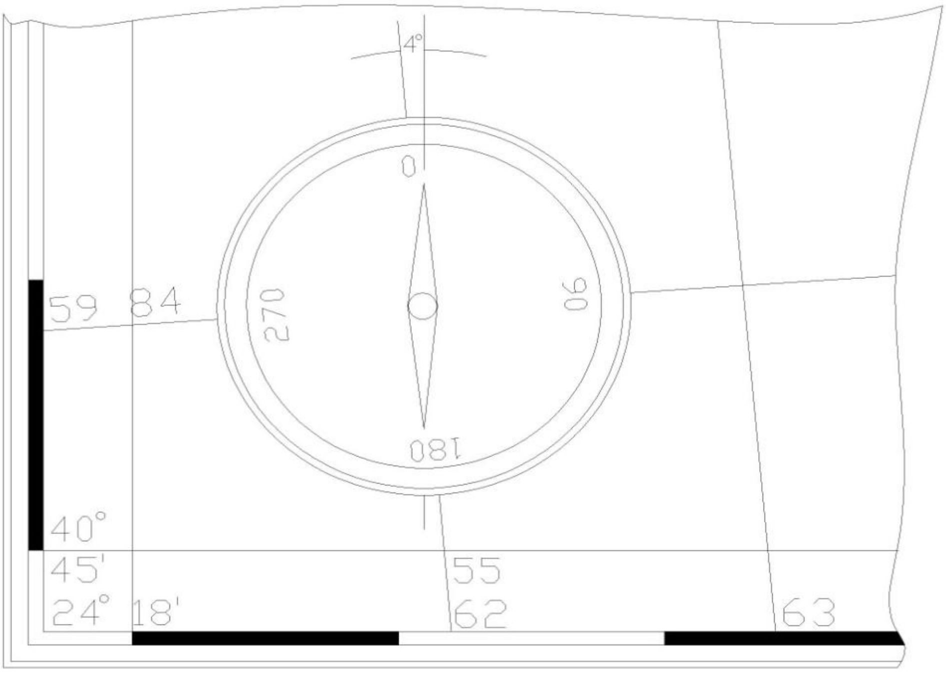


Fig.2.3

can sett up the map so that the zero diameter NS is aligned to the geographic (true) meridian. The map with the compass is rotated until the north end of the needle reads the sum $\gamma + \delta$ =1º45'+2º10'=3º55'~4º that corresponds to the magnetic north of the considered point.

The conventional signs are the characteristically symbols used to express planimetric and relief details on plans and maps.

The conventional signs are classified into two categories: planimetric conventional signs (to scale out of scale and explanatories) and relief conventional signs. Conventional planimetric signs measure depends on the scale we're using (fig.2.4).

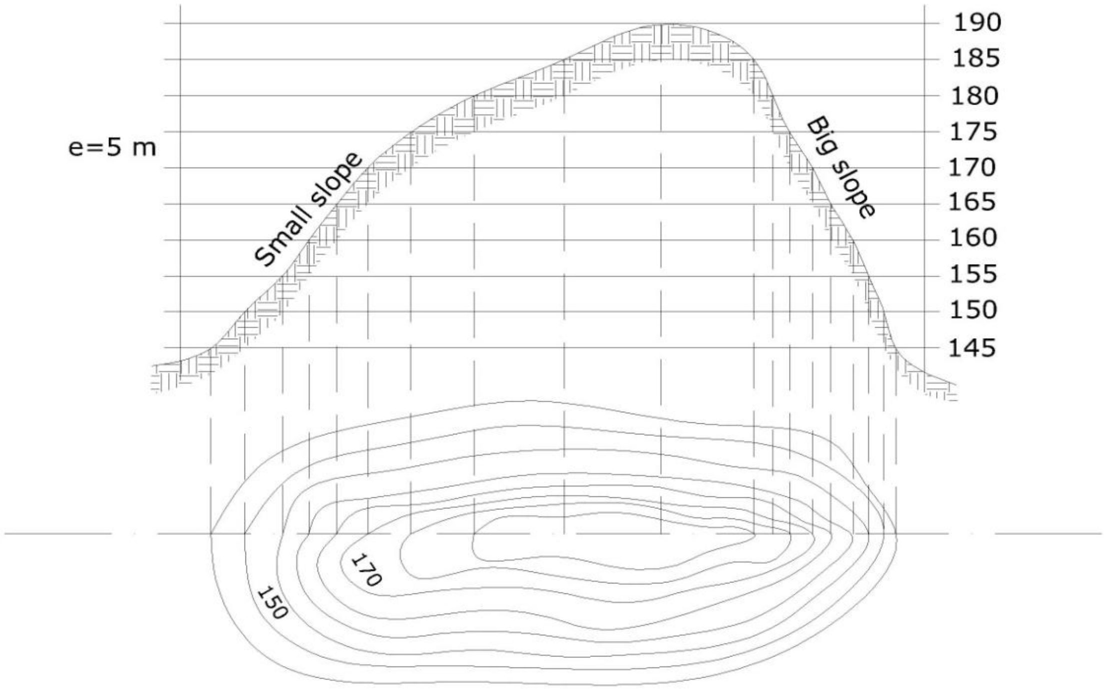


Fig.2.5

The relief of the terrain (level details) is represented on maps and plans using contour lines, hachure’s, hypsometric tints.

The contour line is the horizontal projection of the ground line that unites equal level points. The contour lines can be obtained if we cut the earth’s surface with horizontal planes equally spaced apart (fig.2.5). The vertical distance between two neighboring contour lines is known as the equidistance. Depending on the scale, purpose of plan, etc., the equidistance maybe: 1, 2, 5, 10 m, etc.

The contour lines are of four different kinds: normal, main, secondary, and auxiliary. For certain equidistance the difference between the contour lines on plan is different, depending on the rate of grade (slope) of ground.

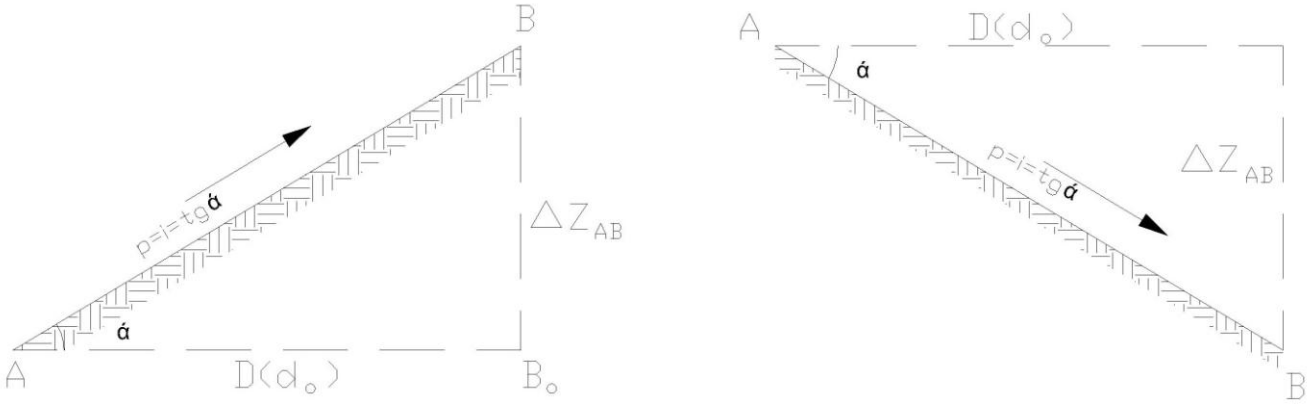


Fig.2.6

The rate of grade (slope) of ground is the tangent, of the slope angle. The Rate of grade of ground between the two points A and B, whose are Z_A and Z_B is noted trough p or I as bellow (fig.2.6).

$$p=i=tg\alpha=\frac{Z_N-Z_M}{D}=\frac{\Delta Z_{MN}}{D}$$

The rate of grade (slope) can be expressed in different ways: in units, per cent or as a ratio. In studying maps, the rate of grade is expressed per cent or in degrees.

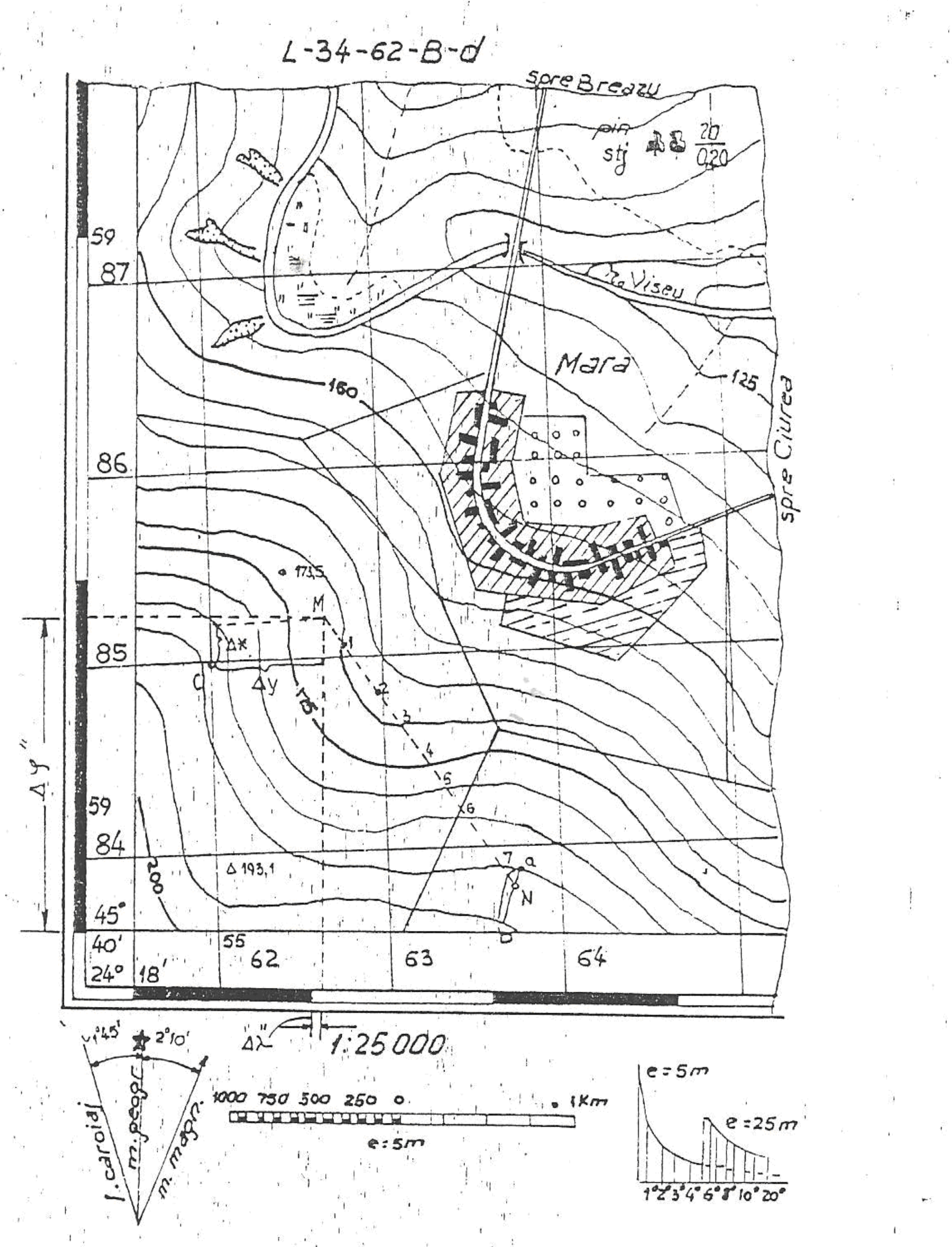
The slope graph is drawn in the leftmost down part of the map and allows us to determinate directly, without calculation, the rate of grade between two contour lines (fig.2.7).

SOLUTION

A. Description of the piece of terrain represented on map relaying on the knowledge of the conventional signs.

Almost in the middle of the surface is the Mara village that is crossed by the road that leads north, towards Breazu village, and eastward, towards Ciurea village. North-east of Mara there’s an orchard and south of Mara we have a vegetable garden. North of Mara the area is crossed by the Viscu River that flows eastwards. In the east there’s an oak and pine-tree forest with trees having a mean height of 20 m and a mean stump of 0.20 m.

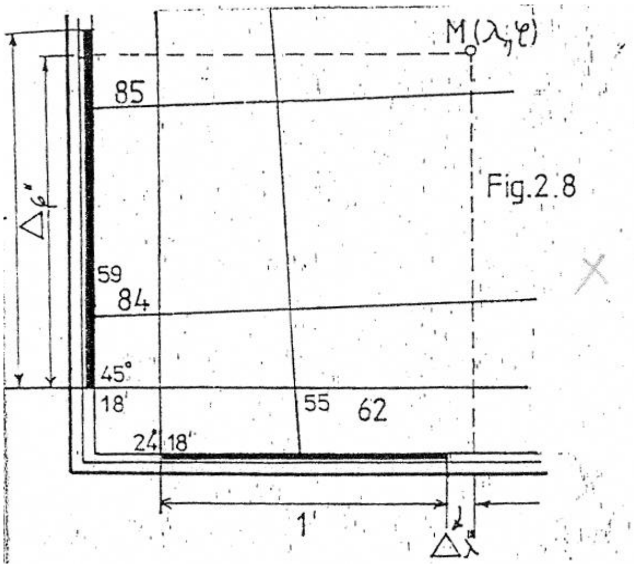
The relief is almost uniform, having level differences within 100 m.

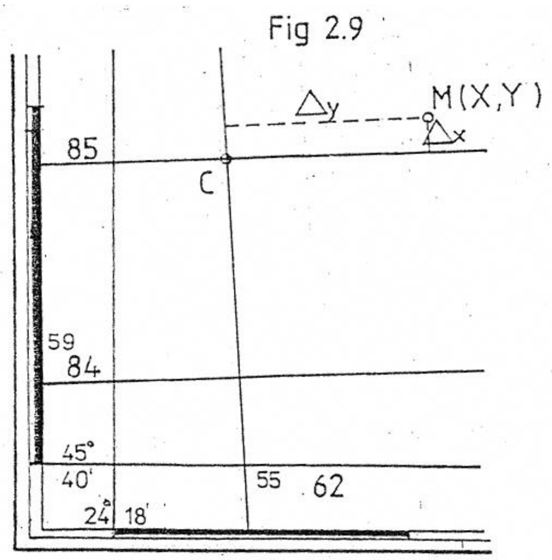


B. Determination of the geographic and plane rectangular coordinates of the M point.

The geographic coordinates of M point, the longitude λ_M, and the latitude, φ_M, are calculated with reference to the map border by linear interpolation, in the following way:

From M point are taken perpendiculars on the map border (fig.2.8) and with reference to these perpendiculars the values of the coordinates are red depending on the values that are written in the border's corner. The lengths corresponding for 1' on longitude and latitude are measured with a ruler, in millimeters, as also the lengths corresponding with the fractions of minute $\Delta\lambda$'' and $\Delta\varphi$''.





The geographic coordinates of point M will be:

λ_M =24º19'+(2,5 mm*60'')/42 mm=24º19'04'' (on horizontal)
φ_M =54º40'+(65 mm*60'')/74 mm=45º40'53'' (on vertical)

The plane rectangular coordinates of point M, the abscissa $X_{M,}$ and the ordinate $Y_{M,}$ are calculated with reference to the lines of the kilometer grid in the following way:

From M point are taken perpendiculars on the grids lines (fig.2.9), and in the same time the coordinates of the south-western corner of the square are determinate, reading the kilometric values inscribed on the border of the map.

X_C =5985 km (on vertical); Y_C = (5) 562 km 9on horizontal)

The number (5) from the ordinate indicates the number of the strip (zone). The distances from the perpendiculars foot to the corner are measuring using a ruler (in millimeters) :

Δ_X =8 mm; Δ_Y =26 mm

According to the scale, we can find out the corresponding distances on ground:

Δ_X = 8 mm*25000=200000 mm=200 m,
Δ_Y =26 mm*25000=650000 mm=650 m,

The plane rectangular coordinates of point M will be:

X_M = 5985000+200 m= 5985200 m
Y_M = (5)562000+650 m= (5)562650 m

C. Level determination of points M and N

When the points are on the contour lines, their level is the same as the ones of the contour lines. When the points are positioned between the contour lines the level determination is made by graphic interpolation.

The level of point M is determinate in the following way:

First we settle the levels of the contour lines between which M is situated, depending on the main contour lines' level and the equidistance. The M point is between the 170 m and 175 m contours lines. Trough M we'll have the line of greatest slope (fig.2.10a) and this line's intersection with the contour lines will be written as a and b.

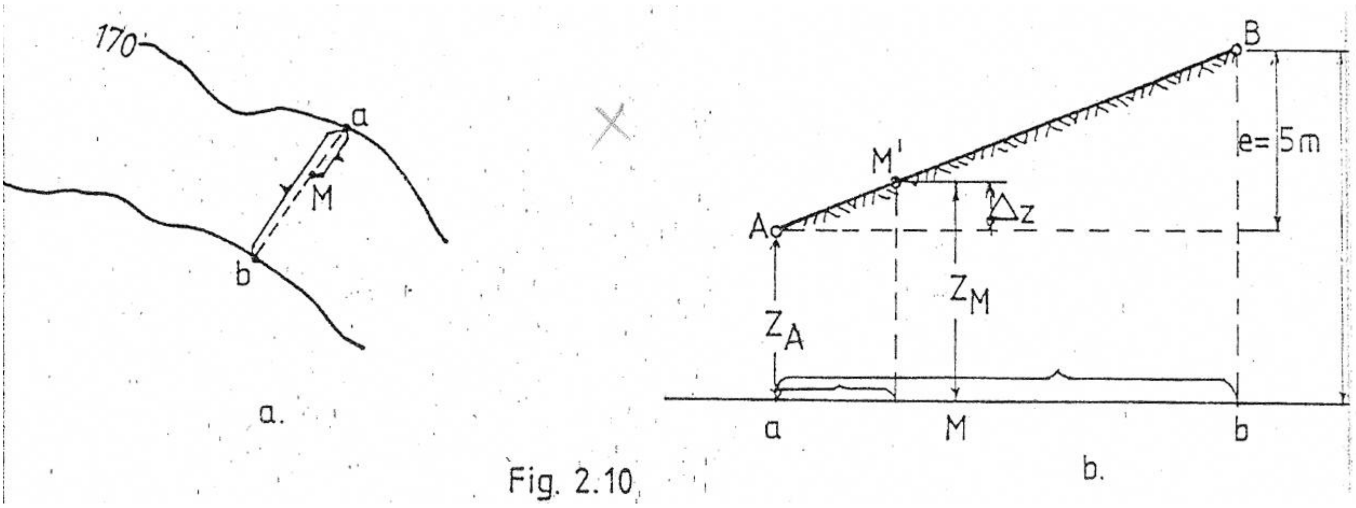


Fig. 2.10

Using the ruler, the segments ab and aM are measured, in millimeters, obtaining for this examples ab=12 mm aM=7,5 mm. from the vertical section (fig.2.10b) the Δ_Z height of the M' point is calculated depending on A point A,B and M' are terrain corresponding points a, b and m from the map).

Δ_Z = (aM/ab)*e= (4.5 mm/12 mm)*5 m=1.9 m

The level for point M is

$Z_M = Z_a + \Delta_Z$ = 170 m+1.9 m =171.9 m

In the same way, for N we found

$Z_N = Z_{A'} + \Delta_{Z'}$ =190 m+1.7 m=191.7 m

D. Determination of the horizontal distance between M and N by graphic measurement of the corresponding line on the map.

The horizontal distance on ground between M and N is calculated depending on its corresponding on the map, measured with a ruler, and the scale's size. From the definition of the numerical scale.

$$\frac{d}{D}=\frac{1}{N}$$

Results that the horizontal distance on ground will be:

D=d*N=71.5 mm*25000=1787500 mm=1787.5 m

E. Determination of rate of grade (slope) of the MN line.

The rate of grade (slope) of MN line is calculated using, the formula (fig.2.6):

$$p=i=tg\alpha=\frac{Z_N\text{-}Z_M}{D}=\frac{\Delta Z_{MN}}{D}$$

The levels of M and N, Z_M and Z_N, have been determinate at point c and the horizontal distance D, at point D. According to them the rate of grade is calculated as a tangent.

P=i=tgα= (191.7-171.9)/1787.5=0.011077

According to this value, the rate of grade (slope) will be expressed in several ways:
-per cent: p%=100*tg =1.108%
-per thousand‰=11.08‰
-as an angular measure: hexadecimal or centesimal:

α^o =arctg (Δ_Z /D) =arctg 0.011077=$0^o38'05''$
α^o =arctg (Δ_Z /D) =arctg 0.011077=$0^g70^c52^{cc}$

F. Laying out a grade line on map between M and N

A constant grade line, also known as the zero level axis, is a tract that is following the terrain according to a required maximum slope, without the necessarily of executing additional equipments.

We have to draw on the map between M and N a grade line with a constant rate of grade (slope) that is p%=1.5%.

The problem consists of finding the map distance d, between two neighboring contour line depending on the required slope. Using the formula for the per cent slope, we'll determinate the ground distance D, that has the required slope p%=1.5% obtaining

$$D=\frac{100\times e}{p\%}=\frac{100\times 5}{1.5}=33.3m$$

Its corresponding on the map is

$$d=\frac{D}{N}=\frac{33.3m}{25000}=0.0133m=13.3mm$$

Laving out the tract of the line on map will be done in the following way (fig.2.11).

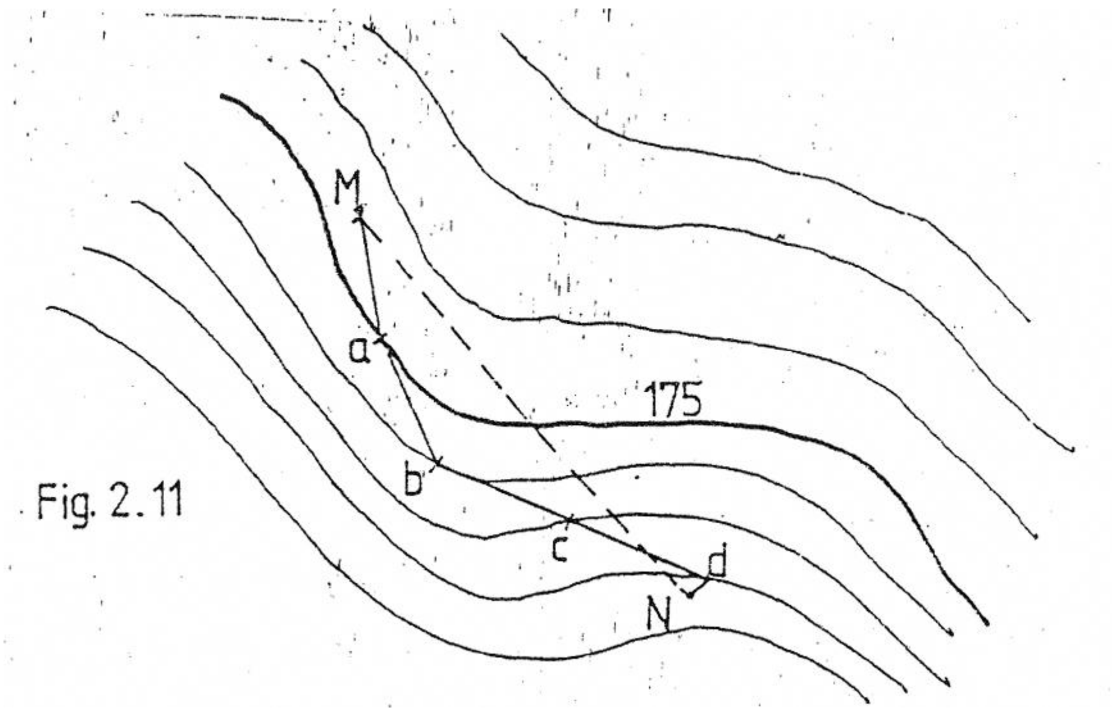


Fig. 2.11

We'll take between the compass's arms a distance d=3.3 mm with the point serving as the center of the circle and we shall intersect the neighboring contour line in point a. With the same span of the compass and in the same way we'll obtain from point a, point b, from b we'll obtain c, from c, d. Uniting these points with a line we shall have the (M-a-b-c-d-N) tract of a required slope p%=1.5%. It is obvious that the slope of M - a and d-N will be smaller than the MN line.

G. Construction the profile of the surface between M and N, considering the length scale for 1/10000 and the elevation scale for 1/1000.

The profile is the line obtained by intersecting earth's surface with a vertical plane that contains the respective direction. For the elaboration ok a topographic profile are used horizontal distances between points and the points' levels.

The elaboration of the ground profile between M and N points on the map is made in the following way.

M and N are united and the intersections with the contour lines are written as 1, 2… 7. (fig.2.7).These points' levels will be equal to the contour lines' levels to witch they belong. We shall draw an arbitrary system of rectangular coordinates taking the horizontal axis as distance scale and the vertical one as the level scale. Usually, the level scale is ten times bigger than the distance scale.

Under the distance scale is drawn a cartouche that contains: the points' number, their levels, the partial and cumulated distances and the rates of grade (slopes) (fig.2.12).Since the distance scale (1:10000)is different to the map's scale (1:25000), the distances measured on plan are first transformed in ground distances and then reduced to the distance scale of the profile. The distances transformation will be done in table 2.1.

Distance cod	Map distance	Corresponding	Distance scale

	1:25000 d(mm)	ground distance D(m)	correspondent 1:10000 d(mm)
M – 1	7.5	187.5	18.75
1 – 2	12	300	30
:	:	:	:
:	:	:	:

Table:2.1

Now, we'll lay off on distance axis the distances between points M, 1, 2… N and we shall fill in the cartouche with the proper elements.

On the level scale (1:1000) will be positioned the levels spaced 5 m apart from the smallest (165 m) to the heights level (195 m).

We'll draw from M, 1, 2…, N points, belonging to the distance scale, perpendiculars, at scale, up to the points' level. Finally, uniting the bottoms of the perpendiculars we will obtain the topographic profile of the terrain along the MN line.

On the slope line is written the rectangle's diagonal in the sense of the slope, and above it is written the rate of grade between two neighboring points. For example, for M – 1, it results

$$p_1\%=\frac{\Delta Z_1}{D_1}=100\times\frac{170-171.9}{187.5}=-1.01\%$$

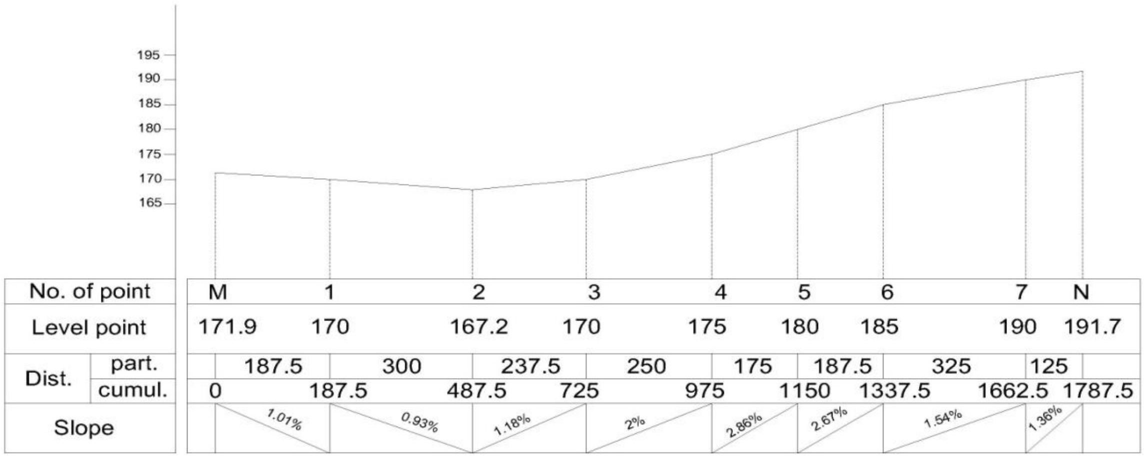
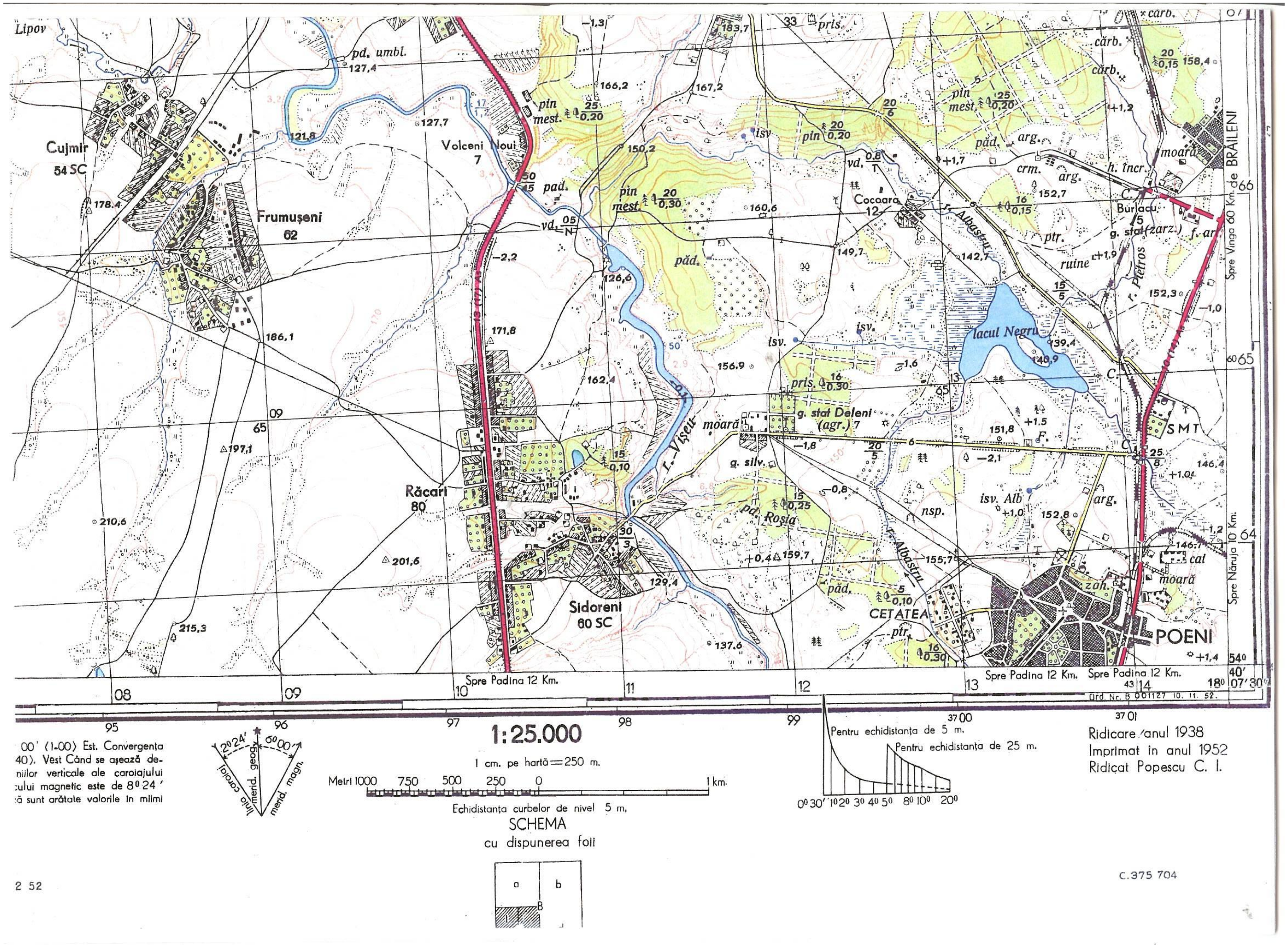


Fig.2.12 Topographic profile between points M and N
Length sc.1:10000 Level sc. 1:1000


Lipov
Cujmir
54 SC
Frumușeni
62
Volceni Noui
7
Răcari
80
Sidoreni
60 SC
Cocoara
12
Burlacu
5
g. stat Deleni
(agr.) 7
CETATEA
POENI
lacul Negru
r. Vișeu
r. Albastru
r. Pietros
isv. Alb
pd. umbl.
moară
Spre Vinga 60 Km. de BRĂILENI
Spre Nănuja 10 Km.
Spre Padina 12 Km.
Spre Padina 12 Km.
Spre Padina 12 Km.
Ord. Nr. 8 001127 10. 11. 52.
1:25.000
1 cm. pe hartă = 250 m.
Metri 1000 750 500 250 0 1 km.
Echidistanța curbelor de nivel 5 m.
SCHEMA
cu dispunerea foii
Pentru echidistanța de 5 m.
Pentru echidistanța de 25 m.
Ridicare anul 1938
Imprimat în anul 1952
Ridicat Popescu C. I.
00′ (1-00) Est. Convergența
40). Vest Când se așează de-
niilor verticale ale caroiajului
ului magnetic este de 8°24′
ă sunt arătate valorile în miimi
merid. geog.
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linia caroiaj
2 52
C.375 704

Application number 3

MEASUREMENTS OF EQUAL OBSERVATIONS

MEASUREMENTS OF EQUAL PRECISION

(EQUAL OBSERVATIONS)

During surveying the equal observations are representing a large volume of work. That's why in the case of equal observations made in order to find out a certain physical quantity it have been elaborated methods of determination of the most reliable value and the evaluation of the precision of the measurements.

Application theme

For finding the distance between two points from the earth's surface a number of equal observations have been made using a steel measuring tape 50 m long. We're interested in the most reliable value of the measured distance and in the evaluation of the precision of the measurements.

Data

The numerical values of the measurements that have been done having as a purpose the determination of the distance between the two points and which are given in column 2 of the 3.1 table .

The application will consist of :

1. Determination of the most reliable value of the measured distance represented by the arithmetic mean.
2. Determination of the accidental errors and the verification [v] = 0.
3. Determination of the squares of the accidental errors and their amount [vv].
4. Determination of the mean square error of a single measurement.
5. Determination of the mean square error of the arithmetic mean.
6. Determination of the relative error of the measured quantity.

SOLUTION

Depending on the results of the equal observations, the operations are carried in the following way:

1. Determination of the most reliable value of the measured distance represented by the arithmetic mean.

Being given the series of measurements in the second column of the 3.1 table, we're going calculate the arithmetic mean.

No. of measure. n	Measured quantities x_i	Arithmetic mean $\overline{X}$	Accidental errors $+/- v_i$	Squares of accident. errors v_i^2	Errors
1	2	3	4	5	6
1	297.51	297.57 9	- 0.069	0.004761	s = + / - 0.045 m s_{x} = + / -0.017 m E_r = + / - 1: 17505
2	297.62		+ 0.041	0.001681	
3	297.58		- 0.001	0.000001	
4	297.60		+ 0.021	0.000441	
5	297.64		+ 0.061	0.003721	
6	297.55		- 0.029	0.000841	
7	297.55		- 0.029	0.000841	
[ ]	2083.05		- 0.005	0.012287	

We are obtaining:

$$x_i = x_i - n * x_2$$

$$\overline{X} = \frac{x_i}{n} = \frac{2083.05}{7} = 297.579 \text{ m}$$

The numerical value of the measured distance is indicated in the third column.

2. Determination of the accidental errors and the verification [v] = 0.

The accidental errors are represented by the algebraic difference, positive or negative, between the results of the measurements and the arithmetic mean .

$$+ / -v_i = x_i - \overline{X} \ , \qquad i= 1,2,.....,7$$

The numerical values are written in column 4. We are going to verify the property according to the algebraic amount of the accidental errors is equal to zero.

$$[\, v_i \,] = + / -v_1 + / - v_2 + / - \ldots\ldots + / - v_7 = - 0.005 \text{ m} \quad 0$$

The amount is going to be written on the last line of the column of the accidental errors.

3. Determination of the squares of the accidental errors and their amount [vv].

In the fifth column are calculated the squares of the accidental errors and on the last line we're going to write their amount.

$$[vv] = v_1^2 + v_2^2 + \ldots.. + v_7^2 = 0.012287$$

4. Determination of the mean square error of a single measurement.

In order to estimate the accuracy of the measurements we shall determine first the mean square error of a single measurement also known as the standard error (STAS 2872 - 74), using the following formula :

$$s = +/- \sqrt{\frac{[VV]}{n-1}} = +/- \sqrt{\frac{0.012287}{7-1}} = +/- 0.045 \text{ m}$$

The standard error is the instrument's error in this case of the 50 m long steel measuring tape. This error is appearing when for the measured distance we would have considered any of the seven results of the measurements. For example, if we would have considered the second measurement ,we would have had :

$$X = x_2 \; +/- \; s = 297.62 \; +/- \; 0.045 \text{ m}$$

5. Determination of the mean square error of the arithmetic mean.

As a statistic indicator for characterizing the dispersion of the arithmetic mean we shall calculate the mean square error by:

$$s_x = +/- \sqrt{\frac{[VV]}{n(n-1)}} = +/- \frac{s}{\sqrt{n}} = +/- \frac{0.045}{7} = +/- \; 0.017 \text{ mm}$$

This is the error that appears in the case that for the measured distance we'll consider the arithmetic mean of the series of measurements.

$$x = \overline{X} \; +/- \; s_x = 297.579 \; +/- \; 0.017 \text{ mm}$$

It is obvious that accepting the arithmetic mean, the error that appears is smaller than before.

6. Determination of the relative error of the measured quantity.

In the case of measuring distances, the errors are growing depending on the values of the lengths. That's why it is necessary the knowledge of the accuracy of measurements for the given length. This accuracy is characterized by the size of the relative error.

The relative error is a ratio between the absolute error and the given quantity and it is expressed in the following way:

$$E_r = \frac{s_x}{\overline{X}} = \frac{1}{\frac{\overline{X}}{s_x}} = \frac{1}{\frac{297.579}{0.017}} = \frac{1}{17504.647}$$

Usually the ratio $R = \frac{\overline{X}}{s_x}$ is rounded off so that the relative error is given as:

$$E_r = \frac{1}{R} = \frac{1}{17505} = 1 : 17505$$

The smaller the relative error the higher the accuracy and inversely.

Generally in surveying the measuring of distances is using steel measuring tapes and measuring tapes ensures an accuracy of 1: 5000 ……….1: 10000 and in geodesy 1: 400000 ……..1: 600000.

Application no.4
THEODOLITES

Theodolites

The theodolite is an instrument used to measure angles, horizontally and vertically. The measurement made using the theodolite are depending on a horizontal plane that passes through the station point and depending on the vertical planes which contain the lines connecting the station point with the ground points.

Application theme:

In order to make some land surveying, which are necessary in projecting and laying out a building, we ought to know how to use the theodolite.

The application will consist of:

1. Indications concerning the scheme of the theodolite and its main parts.
2. Indications in presenting its axis, movements and different types of theodolites.
3. Reading on the graduated circles using an optical device, together with the draw and the way of carrying out the readings.

Solution:

During land surveying, measuring of distances, of horizontal and vertical angles are representing a large volume of work. The theodolite is a goniometric by means which the angular values of the horizontal directions and their inclinations can be measured. According to these values we can determine the vertical and horizontal angles. If the theodolite can measure optically the distances it is called tacheometer. It is thought that:

The ***theodolite*** is an instrument used only in measuring the angular values of the horizontal and inclined directions with high (2cc -10^{cc}) and very high ($0,2^{cc}$-2^{cc}) precision.

They are used to build up the geodetic triangulation network of the country, to developing this network, to lay out construction projects and also to determine the amount of displacements and deformations in an existing structure, in geodesy and in applicated geodesy where we're asked for a high accuracy of the measured angles.

In our country we use the following types of theodolites: *Zeiss-Jena Theo 010,010 A* and *010 B*, *Wild T2, T3* and *T4*, *Kern DKM3*, *MOM TE-B1* etc.

The ***tacheometer*** is used to measure the angular values of the horizontal and inclined directions with a smaller accuracy (20^{cc}-1^{c}) but also at the indirect measuring of distances, optically. These instruments are used in usual surveying measurements, where the required accuracy is smaller. The main types of tacheometers used in our country are: *Zeiss-Jena Theo 030, 020, 020 A, 080, 080 A, Wild* T_1A, T_{16}, *MOM TE-D2, Meopta, Freiberger* etc.

Depending on the way of reading the graduations, theodolites are of different kinds:

- ***<u>Classic theodolites</u>*** (the old type) whose graduated circles, horizontal and vertical, are made of metal and the reading is made using a microscope fixed near these circles.
- ***<u>Modern theodolites</u>***, whose graduated circles are made of glass (crystal) air-tighten covered and the readings are made directly with a special microscope.

1. <u>Indications concerning the scheme of theodolite and its main parts</u>

A representation of classic theodolite is a drawn in ***Fig. 4.1***. The theodolite has the following main parts:

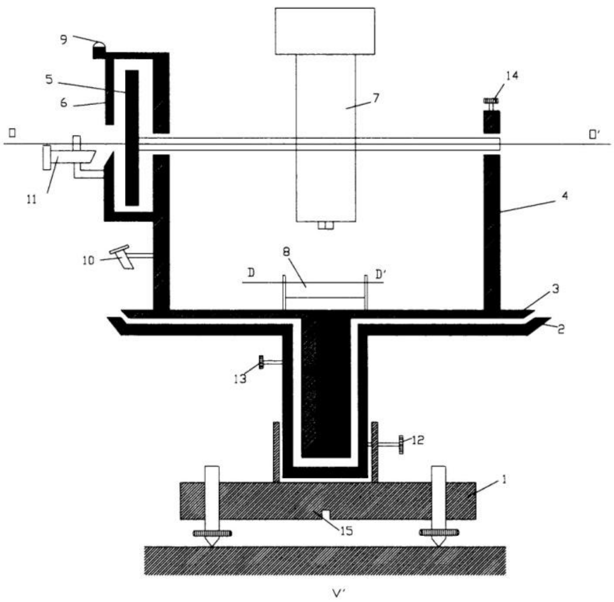
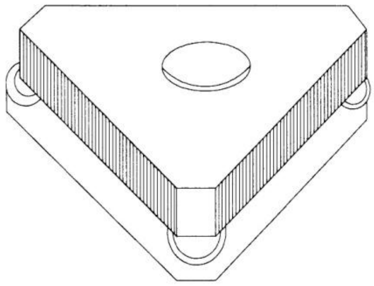
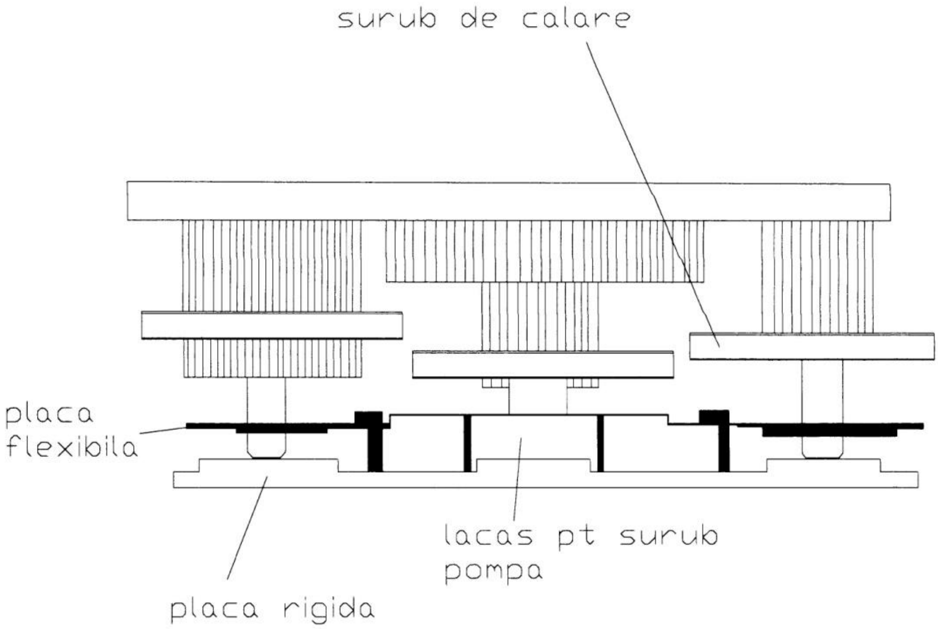


Fig. 4.1 *The parts of theodolites*

- <u>*Three-screw base*</u> (**1**) is a triangular prism that is supported by three leveling screws (***Fig.4.2***).At the bottom, there are two plates: one is the stiff and the other being flexible. The screw that is used to fix the tree-screw base on the tripod, known as instrument-tripod fixing screw is placed in a special place of the flexible plate and the instrument is supported by the stiff plate.



a)



b)

Fig.4.2 *The three-screw base*

- *The graduated horizontal circle* (**2**) is also known as the azimuth circle or limb. This is a metallic disc whose perimeter is silvery and which is engraved with sexazecimal or centesimal degrees (***Fig.4.3 a***). The modern theodolites have a glass made horizontal circle made in the form of a ring with a 50-250 mm diameter fixed on a metallic support (Fig.4.3 ***b).*** The horizontal circle is used in order to read the angular values of the horizontal directions. The movement of the horizontal circle can be stopped using a screw or a level, known as base clamp.

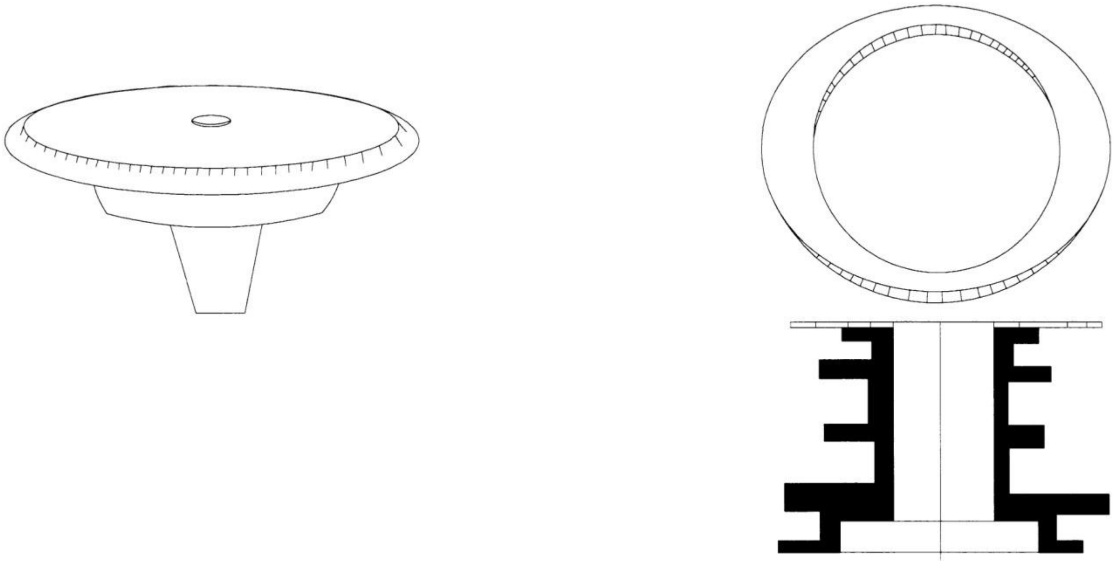


Fig.4.3 *The graduated horizontal circle*

- The upper circle, rotating about the axis passing through the center of the limb is called <u>*vernier plate*</u> (**3**) because it has two opposite verniers, whose zero serves as indexes for reading on the limb. The readings are made using special microscope.The horizontal movement of the vernier plate can be stopped using a screw or a level named as upper clamping screw.
- <u>*The arms for supporting the telescope*</u> (**4**) are metallic and they are fixed an the vernier plate. The arms are supporting the vertical rotation axis of the telescope.
- On one of the arms there is a screw or a lever known as <u>*telescope clamping screw*</u> (**14**), on the other arm there’s a <u>*zenithal level*</u> (**9**) used to bring to the horizontal the indexes of azimuth circle.
- <u>*The telescope*</u> (**7**) is an optical device used to observe the far away objects and to determine the distance by optic way (whenever it has stadia hairs).

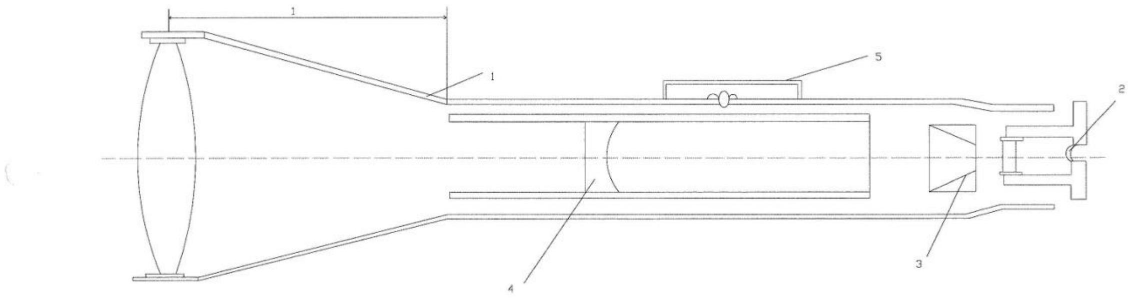


Fig.4.4 *The telescope*

The modern theodolites are equipped with an internal-focusing telescope. This consists of an _objective tube_ (**1**). On their common axis we have the _cross hairs_ (**3**) which is fixed, the _focusing lens_ (**4**) which travels inside the telescope by rotating either a special rack or a _ring_ (**5**) embracing the telescope at the eyepiece. Upon the telescope there is an approximate aiming device known as _collimator_. The image in the case of a telescope is presented in ***Fig.4.5.***

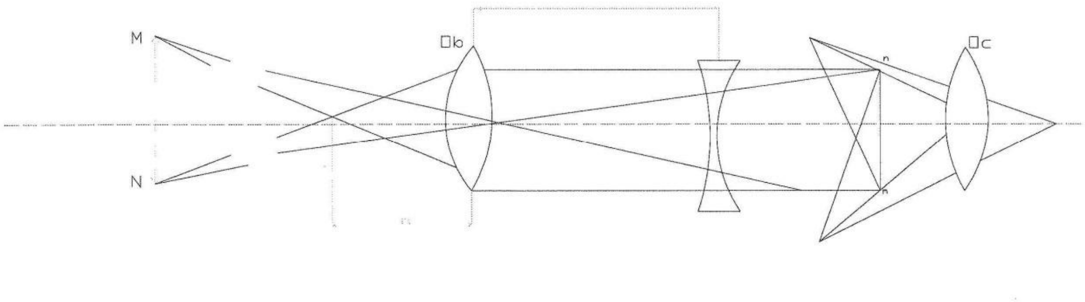
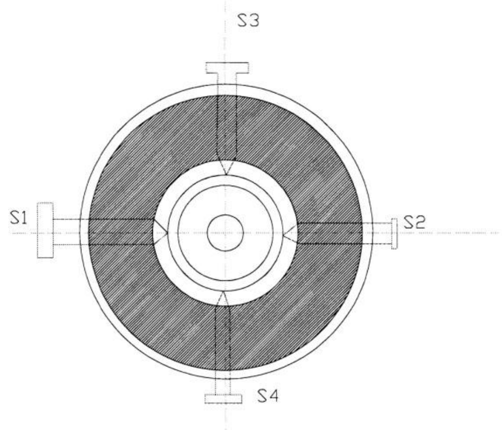


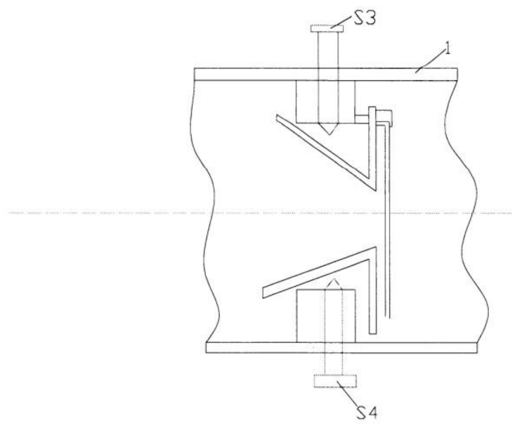
Fig.4.5 *The image in case of a telescope*

The aimed object on the ground, MN, is far from the telescope. The image given by the objective lens, *mn*, is reduced, real and inverted. Passing through the eyepiece, the new image will be *m'n'* and it's going to be virtual enlarged and inverted. The focusing lens (Lf) together with the objective lens are forming a teleobjective whose focal length is greater than the one of the single objective ($l > f_j$). By moving the focusing lens the focal length of the teleobjective is changing so that the image of the aimed object will be clear on the cross hairs plane.

The old theodolites are giving an inverted image of objects which is called _astronomic image_. In order to eliminate this disadvantage the modern theodolites have a lens to restore the disadvantage the modern theodolites have a lens to restore the normal image, obtaining this way a terrestrial image (a straight one).

The _cross hairs_ (**3**) (***Fig.4.6***) consists of glass plate which has engraved two perpendicular line known as _cross hairs_. The glass plate is fitted into a diaphragm aperture with the aid of four screws ***(Fig.4.7***).





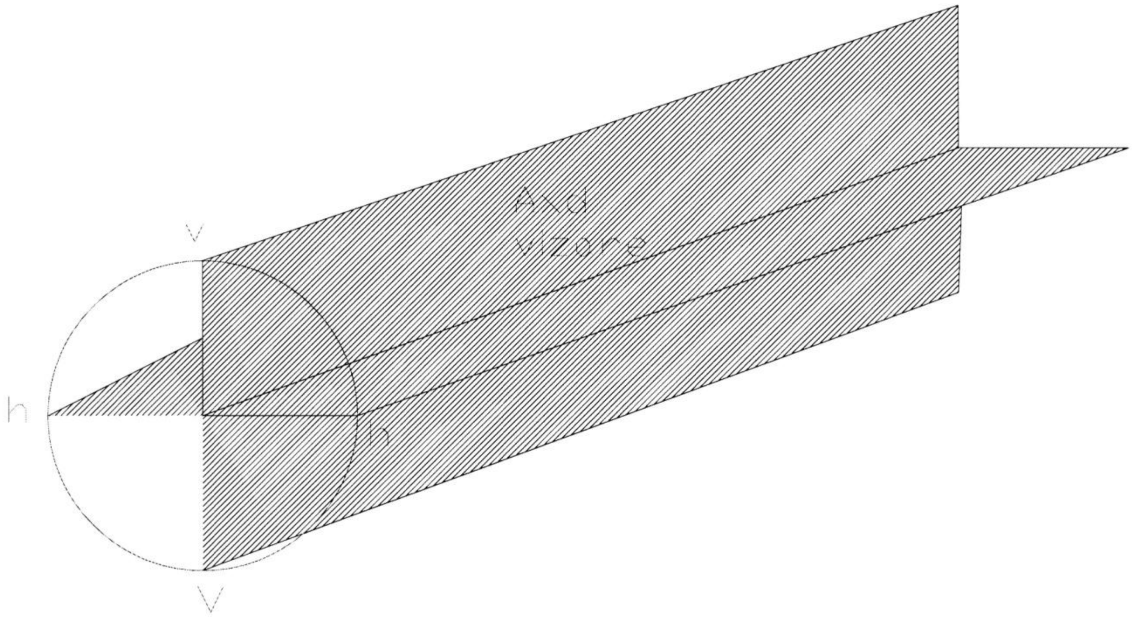
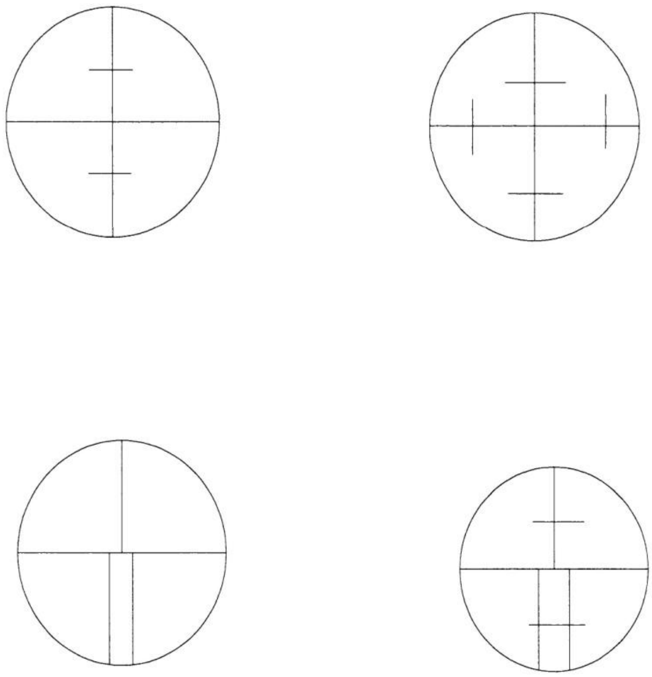


__Fig.4.6__ The cross-hairs



An imaginary straight line passing through the intersection of the cross hairs and through the optical center of the objective is called ***__line of sight__*** (*Fig.4.7*). The tacheometers have two short horizontal parallel and equidistanced lines named as the ***<u>stadia hairs</u>*** and are used in range finding. In *Fig.4.7* there are shown different kinds of cross hairs.

The proper setting of cross hairs is achieved by moving them in a plane perpendicular to the telescope axis with the aid of rectifying screws.

The ***<u>vertical circle</u>*** (**5**) (*Fig.4.1*) is alike with the horizontal graduated circle, rotating about the axis passing through the center of the limb is also called the vernier plate, because it has two opposite verniers, who's zero serves as indexes for reading on the limb.

The upper circle is protected by a fixed circle. The horizontal position of indexes is made by adjustments of the zenithal level using the screw no.7.

At modern theodolites, the reading index (c) in the upper circle is an vertical (*Fig.4.9*).

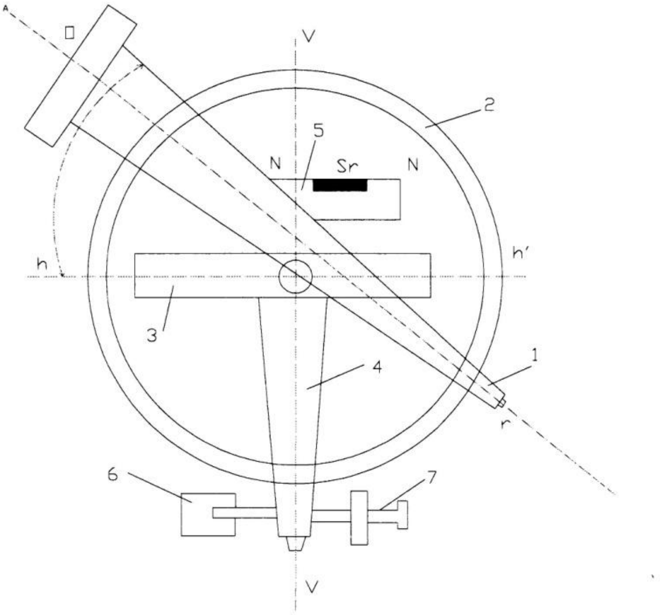


Fig.4.9

The adjustment is being made with a special device (Zeiss-Jena Theo 020, Wild 1 A) a shortly transient mechanical pendulum with air-dubbing and shock-protected supervision.

<u>The level tube</u> (**8**) are special devices used to make some planes or lines horizontal or vertical. Depending on the type we have:

a. *<u>Plane level</u>* (*Fig.4.10 a*) made of a tube of glass which has a special shape filled with alcohol, hermetically and fitted into a metallic support. At the top of level there are several gradations symmetrical to the center point. We also have a bubble of the top of the level. The level is fixed on the superior part of the horizontal circle for adjusting the latter and the arms of the theodolite (zenithal level) to bring into horizontal position the zero indexes on the upper circle.

b. *<u>Circular level</u>* (*Fig.4.10 b*) is fixed on the horizontal circle.

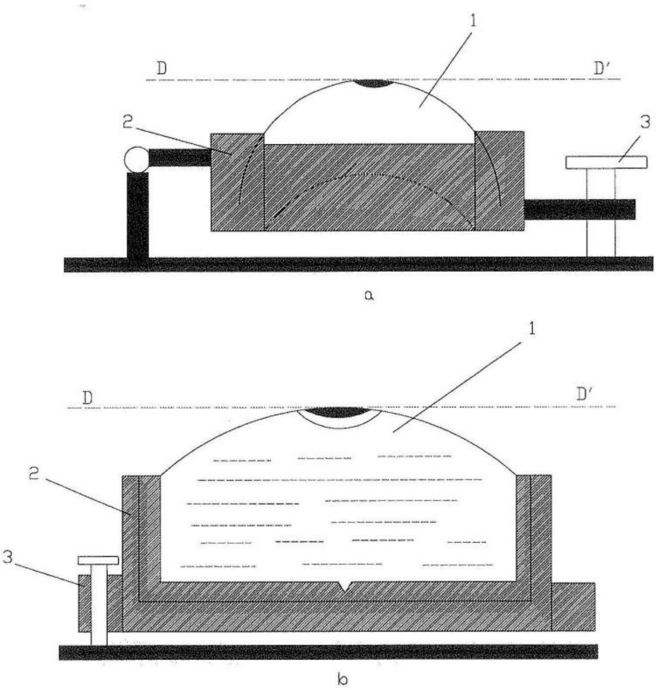


Fig.4.10 *The plane level (a),The circular level (b)*

The main appendages of the theodolite are:

The tripod is a support device that enables the theodolite to be positioned into the station point (*Fig.4.12*). It consists of a small table to which the theodolite is connected with the aid of the tripod fixing screw, its feet being made of wood and whose ends (Fig.4.12) have metallic basses that are driven into ground. The feet of the tripod are telescopic for a better transportation.

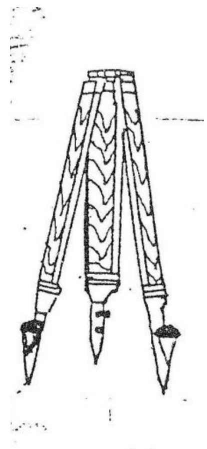


Fig.4.12

The plumb bob consists of a weight with a conical point suspended to a variable length wire (*Fig.4.13 a*). The plumb bob is hanged under the tripod fixing screw making this way the centering of the instrument. The modern theodolites have optical centering devices fixed on the three-screw base inside the theodolite (*Fig.4.13 b*).

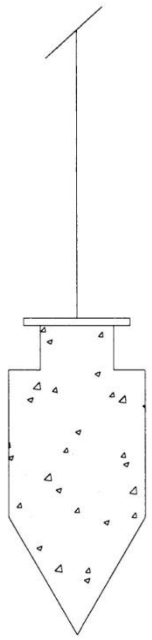


Fig.4.13 The plumb bob

The surveyors compass indicates the magnetic north of the station point. It is situated on the one of the theodolite's arms and ensures measuring of the magnetic bearings of directions, directly on the ground.

2. Indications in presenting its axis, movements and different types of theodolites

The theodolite axes are (Fig.4.14):

a. ***The main axis or the vertical axis*** (V-V') is the axis that is passing through the center of the horizontal angle being perpendicular on the latter. During measurements the main axis has to have the same direction of the vertical of station point.
b. ***The secondary axis or the horizontal axis*** (O-O') is the one passing through the center of the zenithal circle being perpendicular on the latter. It's also known as the rotation axis of the telescope.
c. ***The line of sight*** is a straight line passing through the intersection of the cross hairs and the optical centre of the objective.

These axes have to accomplish several conditions:

- the main axis is perpendicular to the secondary one;
- the line of sight is perpendicular to the secondary one;
- the three axes are intersecting one another in a single point which is known as the mathematical point of the instrument.

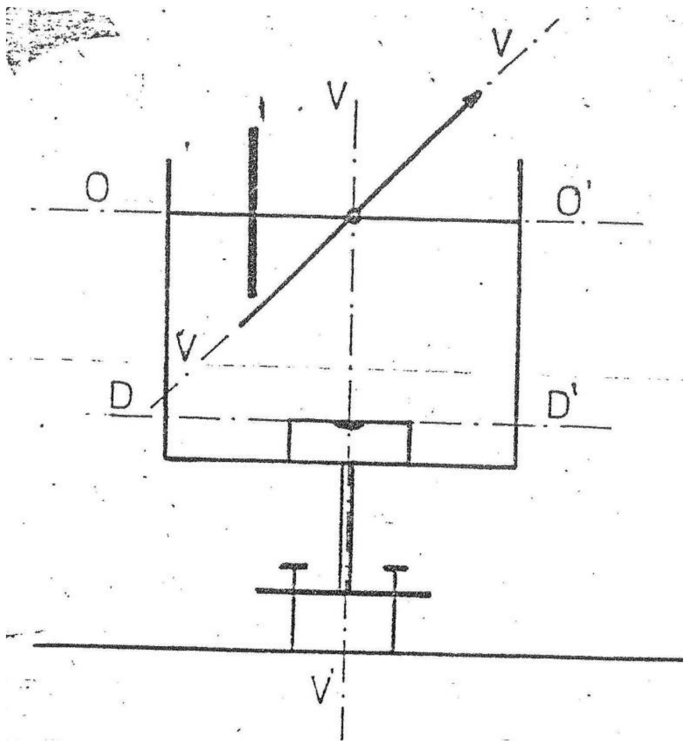


Fig.4.14 *The theodolite axis*

Besides the three axis each plane or circular level has an axis or a direction (D-D'), which after centering has a horizontal position. The theodolite has the following movements:

1. **<u>The horizontal movement</u>** : is the rotation of the instrument around its main axis V-V', in this case there are:
   - ***general movement*** in which the horizontal circle is rotating togheter with the upper circle axis and the reading indexes.
   - ***recording movement*** when the horizontal circle is immobile and only the upper circle is rotating.

The modern theodolites have only a screw or a level to stop the horizontal movement of the theodolite.

2. **<u>The vertical movement</u>** : the telescope and the zenithal circle are moving around the rotation axis (O-O').

The vertical and horizontal movements of the theodolite are controlled by fine motion screws. Depending on the possibilities of moving the vernier plate with reference to the horizontal circle, there are three different kinds of theodolites:

- <u>*Normal theodolites*</u>, whose horizontal circle is immobile by construction, only the vernier plate rotating itself. There are no longer manufactured.
- <u>*Repeating theodolites*</u>, they have a general movement and recoding movement. It's characteristic for the tachometer.

- *Reiterative theodolites* , they have only recording movement and are characteristic for the high precision theodolites.

3. Reading on the gradated circles using its optical devices

The horizontal and vertical circles are divided into grades and minutes. The reading of the fraction of the smallest gradation on the circle is made using the optic reading devices. These can be mechanical (vernier) or optical (the scale microscope). Before reading the gradated circles one should establish:

- type of gradations (sexazecimal or centesimal)
- sense of inscribing the grades (direct or inverse)
- the numerical value of the smallest gradation on the circle (D)
- the precision of reading represented by the value of a gradation of the reading device which is expressed in the following way :

$$p = \frac{D}{n} = \frac{the\ smallest\ gradation\ on\ the\ circle}{number\ of\ gradations}$$

One reading on the gradated circle consists of two parts:

$$C = P_I + P_{II}$$

where:
P_I – the forward reading with reference to the zero index of the reading device
P_{II} - the fraction of gradation read on the reading device

In the case of modern theodolites, whose gradated circles are made of glass, the readings are made using a single microscope positioned on the telescope or on one of the telescope’s arms. Using an optical device (mirrors, lens and prisms), the light rays are view of the microscope.

The index microscope

It is common for tachometers such as Zeiss-Jena Theo 120,080,080 A. On a glass plate which is immobile in the field of the image is superposing on the images of the gradation of the horizontal (Hz) and vertical (V) circles that appear simultaneously in the field of view of the microscope (*Fig.4.15*). The gradations in this case are centesimal, the circles having 400^g and the grades are inscribed clockwise. The smallest gradation from the circles is of 10^c (1^g : 10 grad = 100^c :10 grad = 10^c)

The reading on the horizontal circle is made in the following way :

- the value of grades from the left side of the index is read (207^g) and then the number of integer gradations up to the index is read and multiplied with 10^c, obtaining the first part of the reading $P_I = 207^g 50^c$.

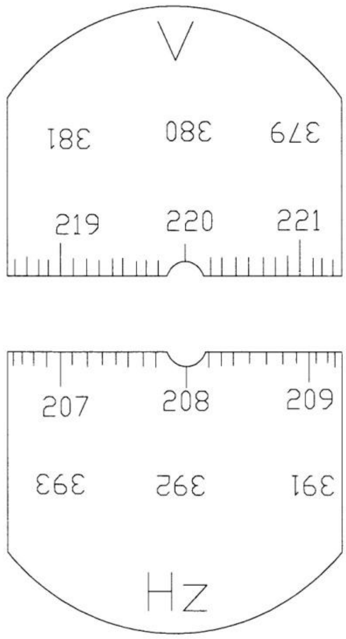


Fig.4.15 *The index microscope*

- the fraction of gradations up to the index, which represents the second part of the reading is estimated with the naked eye and in this case $P_{II} = 9^c$.
- the final reading on the horizontal circle will be:

$$H_Z = P_I + P_{II} = 207^g 50^c + 9^c = 207^g 59^c$$

In the same way the reading on the vertical circle will be:

$$V = P_I + P_{II} = 220^g 30^c + 2^c = 220^g 32^c$$

The scale microscope

It is commonly used at tachometers such as Zeiss-Jena Theo 030,020,020 A, Wild T16. Visible in the microscope's field of view, apart from the image of the circle gradations, there's a scale (*Fig.4.16*).

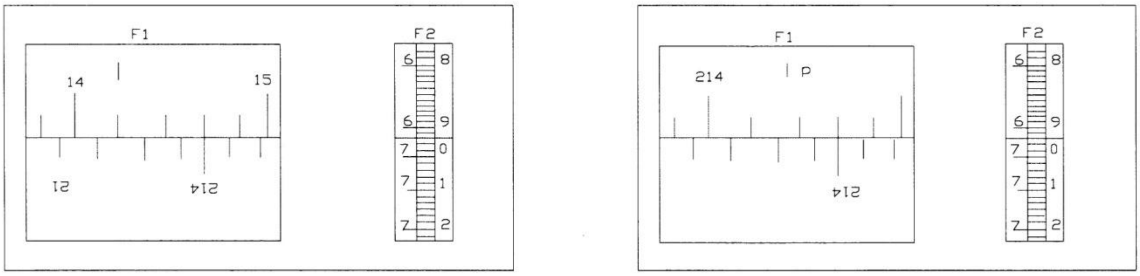


Fig.4.16 *The scale microscope*

In these cases we have two scales divided into 100 divisions which are immobile in the field of view of the microscope. Using an optic device it has been obtained a superposition of the scale image on the gradations of the circles.

By constructing the length of the scale is equal to a gradation from the circle. The circles being divided into grades the accuracy if the device will be:

$$p = \frac{D}{n} = \frac{100^c}{100\ gradations} = 1^c$$

The accuracy represents the numerical value of a gradation on the scale.The reading is made in the following way:

- the first part of the reading consists of gradation circle line that is intersecting the scale: as an example on Hz, $P_I = 271^g$.
- the number of gradations on the scale up to the gradation circle line are counted and multiplied with the accuracy $p = 1^c$, resulting this way the second part of the reading :

$$P_{II} = 93\ gradations\ \cdot 1^c = 93^c$$

- the total reading on the horizontal circle is:

$$H_Z = P_I + P_{II} = 271^g 00^c + 93^c = 271^g 93^c$$

In the same way the reading on the vertical center is:

$$V = P_I + P_{II} = 95^g 00^c + 86^c = 95^g 86^c$$

The two reading devices are used for tachometer whose applications are in usual surveying measurements. Modern theodolites of high accuracy are using digital reading devices with electronic displays.

APPLICATION NB. 6

METHODES OF PLANIMETRIC SURVEY

METHODS OF PLANIMETRIC SURVERY

Relying on the points of the triangulation network or plane surveying network, the planimetric methods allow the developing of the networks in order to obtain the coordinates of some other points and to survey planimetric details from the topographic plane using the method of traverse together with some aiding methods (the method of rectangular coordinates, the method of polar coordinates, the method of bipolar linear measurement results).

Application theme

Having as a purpose the studing and projecting of a commercial center, there has been made the surveying of the considered area using the method of the closed circuit traverse, combined with the method of the polar coordinates. We're asked for the coordinates of the traverse and of the radiation points.

Data

1. The plane rectangular coordinates of the 20 and 22 triangulation points (fig.6.1).

2. The inclined distances, measured directly on the ground using the steel measuring tape, between the points of the traverse and between these points and the radiation points.

3. The angular values of the horizontal directions and the vertical (zenithal) angles of all directions measured from all the traverse points by theodolite.

OBSERVATON: The data of the initial problem (the rectangular coordinates) or the ones obtained by measurements(distances and angles) are entered in a special field book, represented by table 6.1 in the 10,11,3,4 and 5 columns.

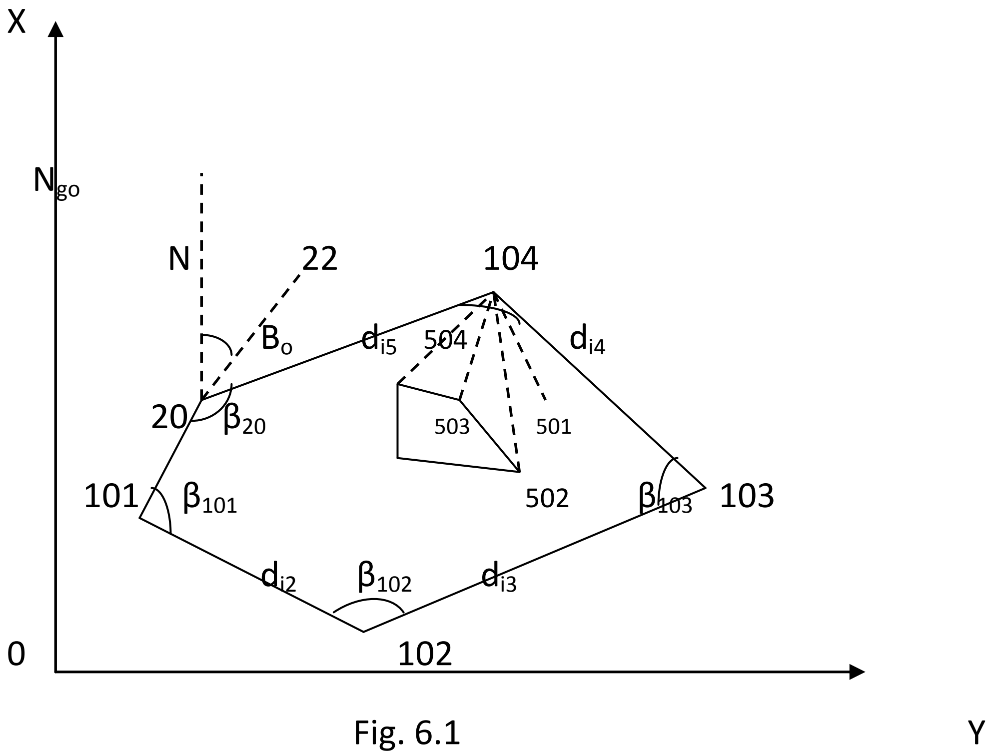


Fig. 6.1

Stat. point	Aimed point	Inclined distances d_i	Zenith. angles Z_i	Horiz. direc. C_i	Absolute coordinates		Point numb.
					X	Y	
1	2	3	4	5	10	11	12
20	22	-	-	0,00	1413,28	2578,44	22
	104	-	-	25,73	1139,75	2111,42	20
	101	89,06	98,07	164,62			101
101	20	-	-	0,00			-
	102	99,77	97,51	103,79			102
102	101	-	-	0,00			-
	103	141,14	98,87	125,97			103

103	102	-	-	0,00			-
	104	91,21	103,70	85,05			104
104	103	-	-	0.00			-
	20	90,84	102,66	146,27			20
	501	30,53	90,17	36,12			501
	502	46,75	93,66	92,63			502
	503	32,25	90,45	112,51			503
	504	43,80	91,79	126,44			504

The application will consist of:

1. Determination of the reference grid bearing using the coordinates of the triangulation points.
2. Determination of the closing error on angles and compensation of the horizontal angles.
3. Transmition of the reference grid bearing to all size of the traverse.
4. Reduction to the horizontal of the relative coordinates of the inclined distances.
5. Determination of the relative coordinates of the traverse points.
6. Determination of the closing error on coordinates and compensation of the relative coordinates.
7. Determination of the absolute coordinates of the traverse points.
8. Determination of the polar and rectangular coordinates of the radiation points.

SOLUTION

During the field operation in order to survey the area there have been made a planimetric closed circuit traverse 20-101-102-103-104-20 , supported by the known point, 20(fig.6.1). For orientating the traverse, from the supporting point, 20, a known triangulation point, 22, has been aimed ,measuring in the same time the horizontal angle, B_0.

The measuring of horizontal angles has been made using the procedure with zero on the initial direction with only one position of the telescope. The zenithal angles have been measured depending on the height of the instrument from the station point. The inclined distances have been measured using a 50 m long steel measuring tape.

The traverse points are given as 101,102,103, and 104, while the radiation points as 501(electric pillar), 502,503,504(the corners of a house).

The office work consist of determination and compensation of the traverse, obtaining thus the rectangular coordinates of the traverse points and then, depending on the latters, the coordinates of the radiation points are being calculated.

The calculation and compensation operations are carried out as follows:

1. Determination of the reference grid bearing using the coordinates of the triangulation points.

$$X_{20} - \frac{n}{2} = X_{20} - 3$$

Knowing the coordinates of the 20 and 22 points, the refence grid bearing, 20-22, is calculated. The relative coordinates are:

$$\Delta x_{20-22=} X_{22-} X_{20} = 1413{,}28 - 1139{,}75 = 273.53\ m$$

$$\Delta y_{20-22=} Y_{22-} Y_{20} = 2578{,}44 - 2111{,}42 = 467.02\ m$$

Since both relative coordinates are positive, the grid bearing is in the first quadrant(see application nb.1). According to fig.6.2, the grid bearing is equal to the calculation angle, β_I , and so,

$$\theta_{20-22=}\beta_{I=arctg}\frac{\Delta y_{20-22}}{\Delta x_{20-22}} = arctg\frac{467{,}02}{273{,}53} = 66^g 27^c 00^{cc}$$

This numerical value is entered in the sixth column.

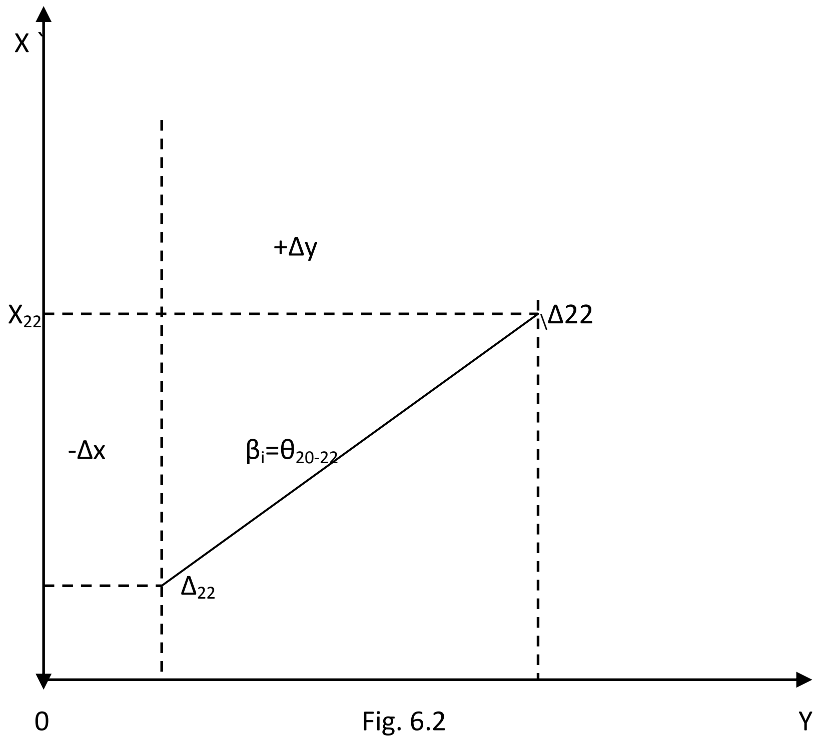
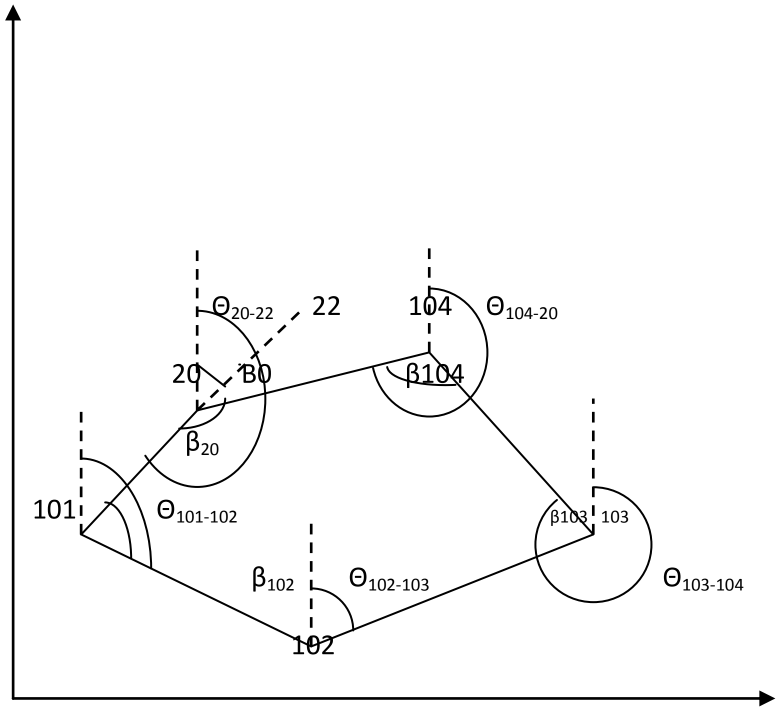


Fig. 6.2



2. Determination of the closing error on angles and the compensation of the horizontal angles.

The internal horizontal angles between the sides of the traverse are obtained as differences of the angular values of the direction, inscribed in the fifth column, so:

$$\beta_{20} = C_{101-}C_{104} = 164^{g}62^{c} - 25^{g}73^{c} = 138^{g}89^{c}$$

$$\beta_{101} = C_{102-}C_{20} = 103^{g}79^{c} - 0^{g}00^{c} = 103^{g}79^{c}$$

$$\beta_{102} = C_{103-}C_{101} = 125^{g}97^{c} - 0^{g}00^{c} = 125^{g}97^{c}$$

$$\beta_{103} = C_{104-}C_{102} = 85^{g}05^{c} - 0^{g}00^{c} = 85^{g}05^{c}$$

$$\beta_{104} = C_{20-}C_{103} = 146^{g}27^{c} - 0^{g}00^{c} = 146^{g}27^{c}$$

The amount of the measured horizontal angles is

$$\sum \beta_I = \beta_{20} + \beta_{101} + \beta_{102} + \beta_{103} + \beta_{104} = 599^g 97^c$$

This amount has to be equal to the amount of the internal angles of the n-sides polygon:

(n - 2) * 200^g= (5 - 2)* 200^g=600^g

Because of the unavoidable instrumental and measuring errors, between the two values there will be a certain difference known as the angular closing error.

$$f_\beta = \sum \beta_I - (n-2) * 200^g = 559^g 97^c - 600 = -3^c$$

As a rule this error has to be smaller then the tolerance.

$|f_\beta| \leq T_\beta$, where $T_\beta = 1^c 50^{cc} * \sqrt{n} = 1^c 50^{cc} * \sqrt{5} = 3^c 35^{cc}$

In this case, the error is smaller then the tolerance, so, by compensation, the correction will be equal in magnitude but with an opposite sign to the one of the closing error.

$$C_\beta = -f_\beta = +3^C$$

The unitary correction is:

$$C_\beta^u = \frac{C_\beta}{n} = \frac{3^c}{5} = 60^{cc}$$

and it is going to be distribuited equally to all the measured angles, obtaining thus the compensated horizontal angles:

$\beta_I^C = \beta_I + C_\beta^u$, i =20,101,102,103,104

$$\beta_{20^C} = 138^g 89^C + 60^{CC} = 138^g 89^c 60^{cc}$$

$$\beta_{101^C} = 103^g 79^C + 60^{CC} = 103^g 79^c 60^{cc}$$

$$\beta_{102^C} = 125^g 97^C + 60^{CC} = 125^g 97^c 60^{cc}$$

$$\beta_{103^C} = 85^g 05^C + 60^{CC} = 85^g 05^c 60^{cc}$$

$$\beta_{104^C} = 146^g 27^C + 60^{CC} = 146^g 27^c 60^{cc}$$

As verification, the amount of compensated angles will be equal to the theoretic amount.

$$\beta_I^C = (n-2) * 200^g = 600^g 00^C 00^{CC}$$

3. Transmition of the reference grid bearing to all sides of the traverse.

This operation is carried out with the aid of the reference grid bearing(θ_{20-22}) and the horizontal angle β_0=$25^g 73^c$ together with the compensated horizontal angles, β_i^C.

Excepting the sides from the first station point, where to the reference, grid bearing we are adding the horizontal angle of the station point

$$\theta_{20-104} = \theta_{20-22} + \beta_0 = 66^g 27^c 00^{cc} + 25^g 73^c 00^{cc} = 92^g 00^c OO^{cc}$$

$$\theta_{20-101} = \theta_{20-104} + \beta_{20^C} = 92^g 00^c 00^{cc} + 138^g 89^c 00^{cc} = 230^g 89^c OO^{cc}$$

starting with the second station point, the rule is: to the previous grid bearing we shall add 200^g to obtain the reverse grid bearing and the horizontal angle from the station point; if the amount outruns 400^g or 800^g, a circle or two will be substract (θ Є [0^g, 400^g] .so,(fig 6.3):

$$\theta_{101-102} = \theta_{20-101} + 200^g + \beta_{101}^C = 230^g 89^c 60^{cc} + 2OO^g + 103^g 79^c 60^{cc}$$
$$= 534^g 69^c 20^{cc} = 134^g 69^c 20^{cc}$$

$$\theta_{102-103} = \theta_{101-102} + 200^g + \beta_{101}^C$$
$$= 134^g 69^c 20^{cc} + 2OO^g + 125^g 97^c 60^{cc} = 460^g 66^c 80^{cc}$$
$$= 60^g 66^c 80^{cc}$$

$$\theta_{103-104} = \theta_{102-103} + 200^g + \beta_{103}^C = 60^g 66^c 80^{cc} + 2OO^g + 85^g 05^c 60^{cc}$$
$$= 345^g 72^c 40^{cc}$$

$$\theta_{104-20} = \theta_{103-104} + 200^g + \beta_{104}^C = 345^g 72^c 40^{cc} + 2OO^g + 146^g 27^c 60^{cc}$$
$$= 692^g 00^C 00^{CC} = 292^g 00^c 00^{cc}$$

The values of the grid bearing are entered in the sixth column.

The verification of this operation is done using the following formula:

$$\theta_{20-104}(forward) = \theta_{104-20}(reverse) +/-200^g$$

so,

$$92^g 00^c 00^c = 292^g 00^c 00^{cc} - 200^g$$

4. Reduction to the horizontal of the relative coordinates of the inclined distances.

Since in the field there have been measured directly the inclined distances, d_i, in the same time the zenithal angles, Z_i, of the directions have been measured. The reduction to the horizontal of the inclined distances is made using the following formula (fig.6.4):

$$d_0 = d_i * \sin Z_i$$

The inclined distances (the third column) are reduced with the aid of the zenithal angles (the fourth column), thus resulting:

$$d_{01} = d_{i1} * \sin Z_1 = 89{,}06 * \sin 98^g 07^c = 89{,}02\, m$$

$$d_{02} = d_{i2} * \sin Z_2 = 99{,}77 * \sin 97^g 51^c = 99{,}69\ m$$

$$d_{03} = d_{i3} * \sin Z_3 = 141{,}14 * \sin 98^g 87^c = 141{,}12\ m$$

$$d_{04} = d_{i4} * \sin Z_4 = 91{,}21 * \sin 103^g 70^c = 91{,}06\ m$$

$$d_{05} = d_{i5} * \sin Z_5 = 90{,}84 * \sin 102^g 66^c = 90{,}76\ m$$

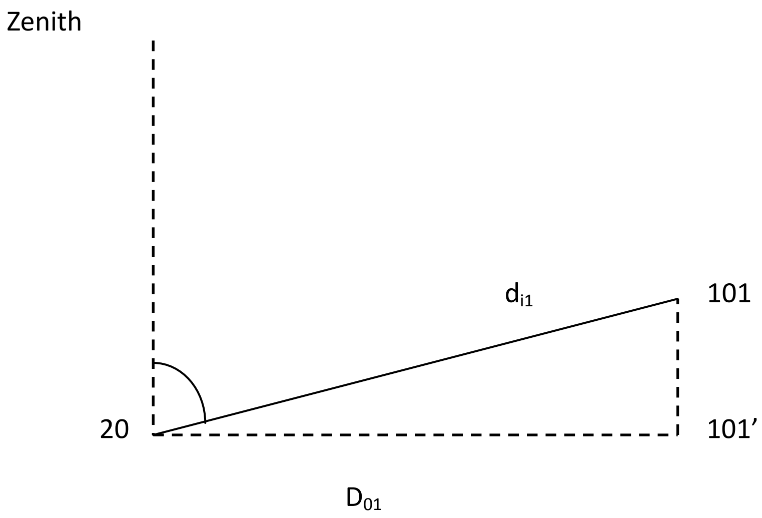


Fig. 6.4

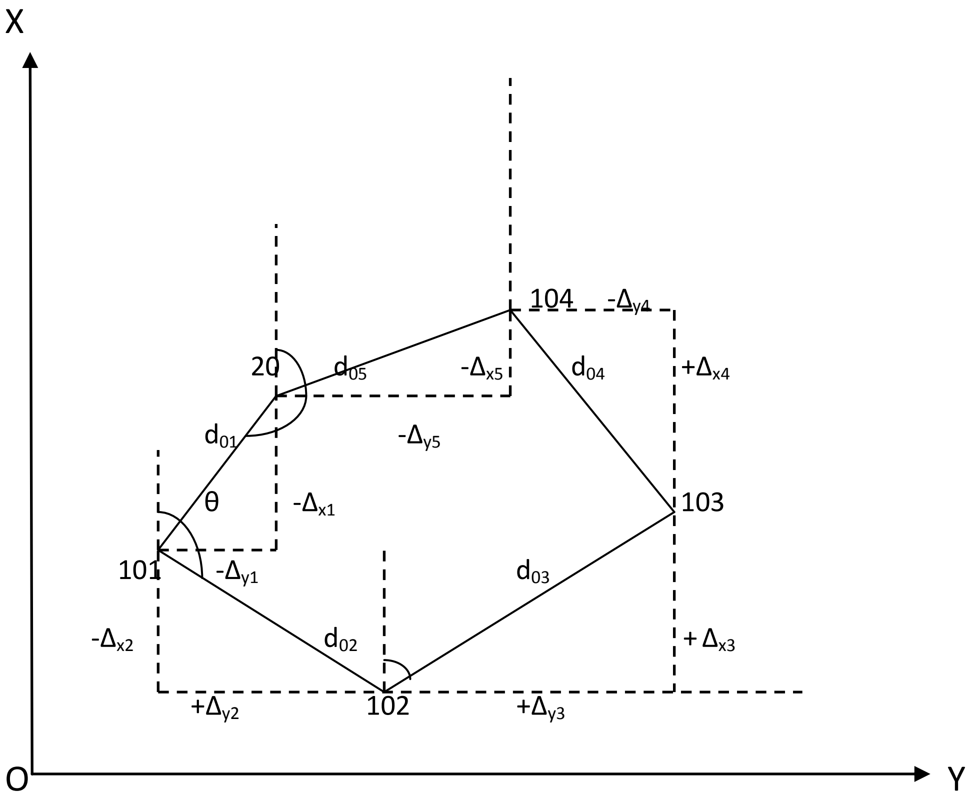


Fig. 6.5

The values of the distances are entered in the seventh column.

5. Determination of the relative coordinates of the traverse points.

The relative coordinates are calculated using the grid bearing of the sides of the traverse and the distances reduced to the horizontal (column 6 and 7 in table 6.2). so,

$$\Delta_x = d_0 * \cos\theta\,; \quad \Delta_y = d_0 * \sin\theta$$

As a convention we are going to use only one index for the relative coordinates so that, as an example, $\Delta_{x_{20-101}} = \Delta_{x_1}$ and$\Delta_{y_{20-101}} = \Delta_{y_1}$. For the five points of the traverse there have been obtained the following relative coordinates:

$$\Delta_{x_1=d_{01}} * \cos\theta_{20-101} = 89{,}02 * \cos 230^g 89^c 60^{cc} = -78{,}74\, m$$

$$\Delta_{y_1=d_{01}} * \sin\theta_{20-101} = 89{,}02 * \sin 230^g 89^c 60^{cc} = -41{,}53\, m$$

$$\Delta_{x_2=d_{02}} * \cos\theta_{101-102} = 99{,}69 * \cos 134^g 69^c 20^{cc} = -51{,}68\, m$$

$$\Delta_{y_2=d_{02}} * \sin\theta_{101-102} = 99{,}69 * \sin 134^g 69^c 20^{cc} = +85{,}25\, m$$

$$\Delta_{x_3=d_{03}} * \cos\theta_{102-103} = 141{,}12 * \cos 60^g 66^c 80^{cc} = +81{,}75\, m$$

$$\Delta_{x_3=d_{03}} * \sin\theta_{102-103} = 141{,}12 * \sin 60^g 66^c 80^{cc} = +115{,}03\, m$$

$$\Delta_{x_4=d_{04}} * \cos\theta_{103-104} = 91{,}06 * \cos 345^g 72^c 40^{cc} = +59{,}92\, m$$

$$\Delta_{x_4=d_{04}} * \sin\theta_{103-104} = 91{,}06 * \sin 345^g 72^c 40^{cc} = +68{,}57 m$$

$$\Delta_{x_5=d_{05}} * \cos\theta_{104-20} = 90{,}76 * \cos 292^g 00^c 00^{cc} = +11{,}38\, m$$

$$\Delta_{x_5=d_{05}} * \sin\theta_{104-20} = 90{,}76 * \sin 292^g 00^c 00^{cc} = +90{,}04\, m$$

The obtained values, rounded off at centimeters, are entered in the eighth and nine columns.

6. Determination of the closing error on coordinates and compensation of the relative coordinates.

Having a closed circuit traverse, the algebraic amounts of the projections of the sides of the traverse on the two axis (i.e. , the algebraic amounts of the relative coordinates) should fulfill the following conditions,

$$\sum_{i=1}^{n} \Delta_{x_i} = 0 \quad and \quad \sum_{i=1}^{n} \Delta_{y_i} = 0$$

but because of the unavoidable measuring errors these conditions will not be accomplished, having the so called "closing error on coordinates" .

$$f_x = \sum_{i=1}^{5} \Delta_{x_i} = -141{,}80 + 141{,}67 = -0{,}13\ m$$

$$f_y = \sum_{i=1}^{5} \Delta_{y_i} = -200{,}14 + 200{,}28 = -0{,}14\ m$$

Having these two errors we can obtain the total error which is:

$$f_L = \sqrt{f_{x^2+} f_{y^2}} = \sqrt{0,13^2 + o,14^2} = 0{,}19\ m$$

Starting from 20, because of the measuring errors, all the points are going to be displaced comparing to the correct position (fig.6.6).

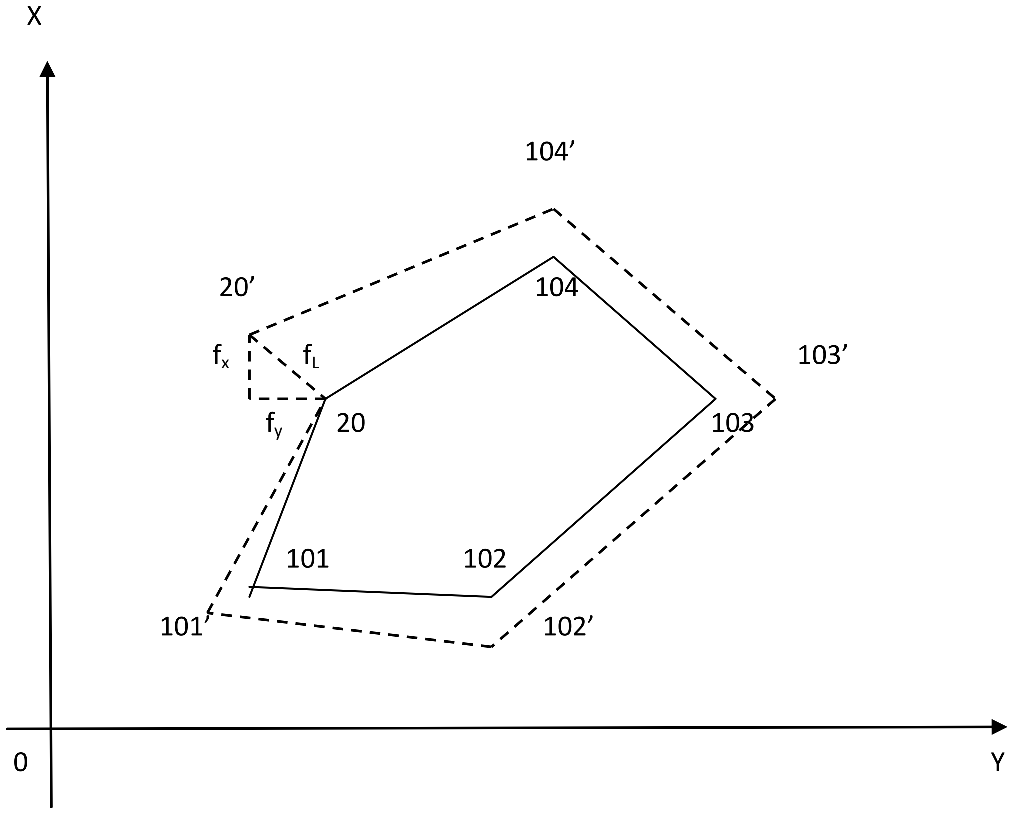


Fig. 6.6

The total error is given by the distances between the displaced point, 20’, and the correct one, 20. In order the measurement to be accepted, the total error has to be smaller than the tolerance. For measurements inside the living areas (cities, towns, villages) the tolerances is:

$$T = 0{,}003 * \sqrt{D} + \frac{D}{2600}$$

For the considered case, the length of the traverse is $D=d_{o1}+d_{o2}+d_{o3}+d_{o4}+d_{o5}$= 511.65 m, so that the tolerance is:

$$T = 0{,}003 * \sqrt{511{,}65} + \frac{511{,}65}{2600} = 0{,}27\, m$$

The error is accepted since it is smaller than the tolerance.

$$f_L \leq T;\ 0{,}26m < 0{,}27m$$

The next operation is the one of compensating the traverse, the relative coordinates being compensated, bringing the points of the traverse from their displaced positioned into the correct ones.

The corrections are equal in magnitude and opposite in sign to the errors:

$$C_x = -f_x = +0{,}13\, m\, ;\; C_y = -f_y = -0{,}14\, m$$

The distribution of the corrections is made proportional to the value of the relative coordinates. The corrections per meter are:

$$C_x^u = \frac{C_x}{\sum_{i=1}^{5}\left|\Delta_{x_i}\right|} = \frac{+0{,}13}{283{,}47} = +0{,}000423\ {}^{m}/_{m}$$

$$C_y^u = \frac{C_y}{\sum_{i=1}^{5}\left|\Delta_{y_i}\right|} = \frac{+0{,}14}{400{,}42} = -0{,}000349\ {}^{m}/_{m}$$

and according to them, the partial corrections for each of the relative coordinate is determinate as:

$$C_{\Delta_{x_i}} = C_x^u * \left|\Delta_{x_i}\right|\, ;\; C_{\Delta_{y_i}} = C_y^u * \left|\Delta_{y_i}\right|$$

As an observation, the signs of the partial corrections are the same with the sings of the total corrections. There have been obtained the following value:

$$C_{\Delta_{x_1}} = C_x^u * \left|\Delta_{x_1}\right| \approx 0{,}03\, m\ ;\; C_{\Delta_{y_1}} = C_y^u * \left|\Delta_{y_1}\right| \approx -0{,}01\, m$$

$$C_{\Delta_{x_2}} = C_x^u * \left|\Delta_{x_2}\right| \approx 0{,}02\, m\, ;\; C_{\Delta_{y_2}} = C_y^u * \left|\Delta_{y_2}\right| \approx -0{,}03\, m$$

$$C_{\Delta_{x_3}} = C_x^u * \left|\Delta_{x_3}\right| \approx 0{,}04\, m\ ;\; C_{\Delta_{y_3}} = C_y^u * \left|\Delta_{y_3}\right| \approx -0{,}05\, m$$

$$C_{\Delta_{x_4}} = C_x^u * \left|\Delta_{x_4}\right| \approx 0{,}03\, m\ ;\; C_{\Delta_{y_4}} = C_y^u * \left|\Delta_{y_4}\right| \approx -0{,}02\, m$$

$$C_{\Delta_{x_5}} = C_x^u * \left|\Delta_{x_5}\right| \approx 0{,}01\, m\ ;\; C_{\Delta_{y_5}} = C_y^u * \left|\Delta_{y_5}\right| \approx -0{,}03\, m$$

As a verification, the amounts of the partial corrections have to be equal to the total corrections.

$$\sum_{i=1}^{5} C_{\Delta_{xi}} = C_x = +0,13\,m\,;\ \sum_{i=1}^{5} C_{\Delta_{yi}} = C_y = -0,14\,m$$

By algebraic amount of the previously calculated relative coordinates with the corresponding partial corrections, the compensated relative coordinates will be obtained as:

$$\Delta_{xi}^{C} = \Delta_{xi} + C\Delta_{xi}\,;\ \Delta_{yi}^{C} = \Delta_{yi} + C\Delta_{yi}$$

so,

$$\Delta_{x1}^{C} = -78,74 + 0,03 = -78,71\,m\,;\ \Delta_{y1}^{C} = -41,53 - 0,01 = -41,54\,m$$

$$\Delta_{x2}^{C} = -51,68 + 0,02 = -51,66\,m\,;\ \Delta_{y2}^{C} = +85,25 - 0,03 = +85,22\,m$$

$$\Delta_{x3}^{C} = +81,75 + 0,04 = +81,79\,m\,;\ \Delta_{y3}^{C} = +115,03 - 0,05 = +114,98\,m$$

$$\Delta_{x4}^{C} = +59,92 + 0,03 = +59,95\,m\,;\ \Delta_{y4}^{C} = -68,57 - 0,02 = -68,59\,m$$

$$\Delta_{x5}^{C} = -11,38 + 0,01 = -11,37\,m\,;\ \Delta_{y5}^{C} = -90,04 - 0,03 = -90,07\,m$$

After compensation, the algebraic amounts of the relative coordinates will be equal to zero:

$$\sum_{i=1}^{5} \Delta_{xi}^{C} = -141,74 + 141,74 = 0\,;\ \sum_{i=1}^{5} \Delta_{yi}^{C} = -200,00 + 200,00 = 0$$

In table 6.2, in the eighth and nein columns there have been entered the initial relative coordinates, the corrections (in centimeters) and the compensated coordinates.

7. Determination of the absolute coordinates of the traverse points.

Using the absolute coordinates of the supporting point, 20, we shall add, step by step and cumulated, the compensated relative coordinates, thus obtaining the absolute coordinates of all station points.

$$X_{101} = X_{20} + \Delta_{x1}^{C} = 1139{,}75 - 78{,}71 = 1061{,}04\, m$$

$$X_{102} = X_{101} + \Delta_{x2}^{C} = 1061{,}04 - 51{,}66 = 1009{,}38\, m$$

$$X_{103} = X_{102} + \Delta_{x3}^{C} = 1009{,}38 + 81{,}79 = 1091{,}17\, m$$

$$X_{104} = X_{103} + \Delta_{x4}^{C} = 1091{,}17 + 59{,}95 = 1151{,}12\, m$$

$$X_{20} = X_{104} + \Delta_{x5}^{C} = 1151{,}12 - 11{,}371 = 1139{,}75\, m$$

$$Y_{101} = Y_{20} + \Delta_{y1}^{C} = 2111{,}42 - 41{,}54 = 2069{,}88\, m$$

$$Y_{102} = Y_{101} + \Delta_{y2}^{C} = 2069{,}88 + 85{,}22 = 2155{,}10\, m$$

$$Y_{103} = Y_{102} + \Delta_{y3}^{C} = 2155{,}10 + 114{,}98 = 2270{,}08\, m$$

$$Y_{104} = Y_{103} + \Delta_{y4}^{C} = 2270{,}08 - 68{,}59 = 2201{,}49\, m$$

$$Y_{20} = Y_{104} + \Delta_{y5}^{C} = 2201{,}49 - 90{,}07 = 2111{,}42 m$$

As a verification, the final coordinates of 20 will be equal to the initial ones. The data are entered in the tenth and eleventh columns, table 6.2.

8. Determination of the polar and rectangular coordinates of the radiation points.

The way of determining of plane position of the radiation points is made depending on the nature of the details. Thus, the details that have no precise limits (lakes, rivers, forest borders) are determinate by polar coordinates (β, d_o), while characteristic points and precise limits (buildings, streets, bridges) are determinate by rectangular coordinates (X, Y).

The radiation points are determinate with reference to the traverse points have been measured.

a) Determination of the polar coordinates. The 501 point representing an electric pillar will be determinate by polar coordinates: the horizontal angels, β_{501} ,measured on the ground with reference to the previous side of the traverse and which is entered in the fifth column, and the distance reduced to the horizontal, d_0, which is calculated with the aid of the inclined distance, d_i, and the zenithal angle, Z. Point 501(β_{501}=36^g12^c;d_o=d_i *sin Z=30,53* sin 90^g17^c=30,17 m)

b) Determination of the rectangular coordinates. The 502, 503 and 504 points, representing the corners of a house, will be determinate by rectangular coordinates. For one of the points, for example 502, the calculation is carried out as follows:

- determination, by transmition, of the 104-502 grid bearing, in the same way as in the case of the close traverse.

$$\theta_{104-502} = \theta_{103-104} + 200^g + \beta_{502} = 345^g 72^c 40^{cc} + 200^g + 92^g 63^c$$
$$= 638^g 35^c 40^{cc} = 238^g 35^c 40^{cc}$$

- determination of the distance reduced to the horizontal

$d_{0,502} = d_{i,502} * \sin Z_{502} = 46{,}75 * \sin 93^g 66^c = 46{,}52$ m

- determination of the relative coordinates of 502 with reference to the 104 traverse point.

$$\Delta_{x_{104-502}} = d_{0,502} * \cos \theta_{104-502} = 46.52 * \cos 238^g 35^c 40^{cc} = -38{,}33\ m$$

$$\Delta_{y_{104-502}} = d_{0,502} * \sin \theta_{104-502} = 46.52 * \sin 238^g 35^c 40^{cc} = -26{,}36\ m$$

- determination of the absolute coordinates of the 502 radiation point, adding relative coordinates just obtained to the absolute coordinates of the 104 traverse point, so

$X_{502} = X_{104} + \Delta_{x_{104-502}}$=1151,12-38,33=1112,79 m

$Y_{502} = Y_{104} + \Delta_{y_{104-502}}$=2201,49-26,36=2175,13 m

The resulting values of the coordinates of the 502 radiation point are entered in table 6.2 in the 6, 7, 8, 9, 10 and 11 columns on the line of the 502 point.

In the same way the coordinates of the 503 and 504 radiation points have been calculated.

Stat. point	Aimed. point	Inclined distances d_i	Zenith. angles Z_i	Horiz. direc C_i	Grid bearings θ	Distances reduced to horizontal d_0	Relative coordinates		Absolute coordinates		Point numb.
							$+/-\Delta_x$	$+/- \Delta_y$	X	Y	
1	2	3	4	5	6	7	8	9	10	11	12
20	22	-	-	0,00	$66^g.27^c.00^{cc}$	-	-	-	1413,28	2578,44	22
	104	-	-	25,73	92.00 00	-	-	-	1139,75	2069,88	20
	101	89,06	98,07	164,62	230.89 60	89,02	-78,71	-41,54	1061,04	-	101
101	20	-	-	0,00	-	-	-	-	-	2155,10	-
	102	99,77	97,51	103,79	134.69 20	99,69	-51,66	85,22	1009,38	-	102
102	101	-	-	0,00	-	-	-	-	-	2271,08	-
	103	141,14	98,87	125,97	60.66 80	141,12	81,79	114,98	1091,17	-	103
103	102	-	-	0,00	-	-	-	-	-	2201,49	-
	104	91,21	103,70	85,05	345.72 40	91,06	59,95	-68,59	1151,12	-	104
104	103	-	-	0,00	-	-	-	-	-	2111,42	-
	20	90,84	102,66	146,27	292.00 00	90,76	-11,37	-90,07	1139,75	-	20
	501	30,53	90,17	36,12	181.84 40	30,17	-	-	-	-	501
	502	46,75	93,66	92,63	238.35 40	46,52	-38,33	-26,36	1112,80	2175,14	502
	503	32,25	90,45	112,51	258.23 40	31,89	-19,45	-25,27	1131,67	2176,23	503
	504	43,80	91,79	126,44	272.16 40	43,02	18,22	38,97	1132,81	3163,43	504

Application no.7

DRAWING UP

TOPOGRAPHIC PLANS

DETERMINATION OF AREAS

The application will consist of:
1.Drawing up the topographic plan to a scale of 1:1000
2.Determination of the area inside the closed circuit traverse by graphic metode, by two divisions in simple geometric shapes, together with the verification of the two divisions and establishing of the final result.
3.Determination of the area using the mechanic method, using the polar planimeter, including the scale constant determination.
4.Area determination by analytic method.

SOLUTION

According to the linear and angular measurement results, the calculation and compensation of the planimetric survey has been made together with the determination of the characteristic points of the planimetric details, resulting the rectangular and polar coordinates of all points. Relying on the latters, the drawing up of the topographic plan and area determination has been made.

1. Drawing up the topographic plan to a scale of 1:1000

Drawing up the topographic plans can be done by precise methods using the plane rectangular coordinates(X, Y) or by approximative methods using the polar coordinates (B, d_0), or, usually, by combined methods using both rectangular and polar coordinates. Drawing up topographic plans takes a few stages:

a.Preparatory stage. During this stage the inventary of coordinates of all points to be plotted is made, the paper and drawing instruments are prepared, the scale of the map is chosen and also the coordinates system is established.

Drawing up topographic plans using scales smaller than 1:2500(as example 1:5000, 1:10000) is made by stereographic projection 1970 on metal mounted paper. On this paper is established the format and the axis of coordinates after which the map border is plotted by rectangular coordinates, the index of the map being inscribed also.

Inside the map border the ground points are plotted. Drawing up site plans is made on scale paper on which is established the format and the axis of coordinates, but in this case the map border is no longer draw.

The format is determinate in the following way:
from the inventary of coordinates(table 7.1), the maximum and minimum values of the abscises and ordinates are considered, making at the same time the following differences:

$\Delta X=X_{max}-X_{min}=X_{104}-X_{102}$ 142m

$\Delta Y=Y_{max}-Y_{min}=Y_{103}-Y_{101}$ 200m

To this values, of course reduced at scale(1:1000) of the plan, there are added 10. . . 20 cm, obtaining thus the length(20+15=35cm)and the width(14+15=29cm) of the format. The instruments used in drawing up topographic plans are: the rectangular coordinatograph , the protractor, the ruler, the set square, the compass and drawing pens.

After setting the paper, the axis of coordinates are established. As origin of the rectangular system of coordinates are chosen two integer values, X_0 and Y_0, smaller than the smallest abscisa and ordinate from the inventary of coordinates. This way,

$X_0<X_{min}=X_{102}=1009.38$, so $X_0=1000$m

$Y_0<Y_{min}=Y_{101}=2069.88$, so $Y_0=2050$m

This thing is done in order to ensure that all points will be able to be plotted in the chosen system of axis .

The next operation is drawing the square grid, considering on the two axis points 50, 100, 200, 500 or 1000m spaced apart, depending on the scale of the plan. 1:1000 being a large scale these points are 50m spaced apart. From these points are taken perpendiculars, obtaining in this way the square grid of the plan.

b.Plotting the points. The operation of plotting points on plan by their rectangular coordinates(triangulation, traverse or radiation points) is achieve with reference to the south-western corner of the square in which the points are positioned. So, the triangulation point 20(1139.75; 2111.42) is plotted with reference to the corner of the square that has the coordinated X=1100 and Y=2100(fig.7.1 and fig.7.2), by measuring with a ruler the relative coordinates ΔX=1139.75-1100=39.75m(at scale ΔX=39.75m≈40m) and ΔY=2111.42-2100=11.42m(at scale ΔY=11.42m≈11.5m). From the two points are taken perpendiculars, their intersection representing the plane position of point 20. In the same way the other points are plotted.

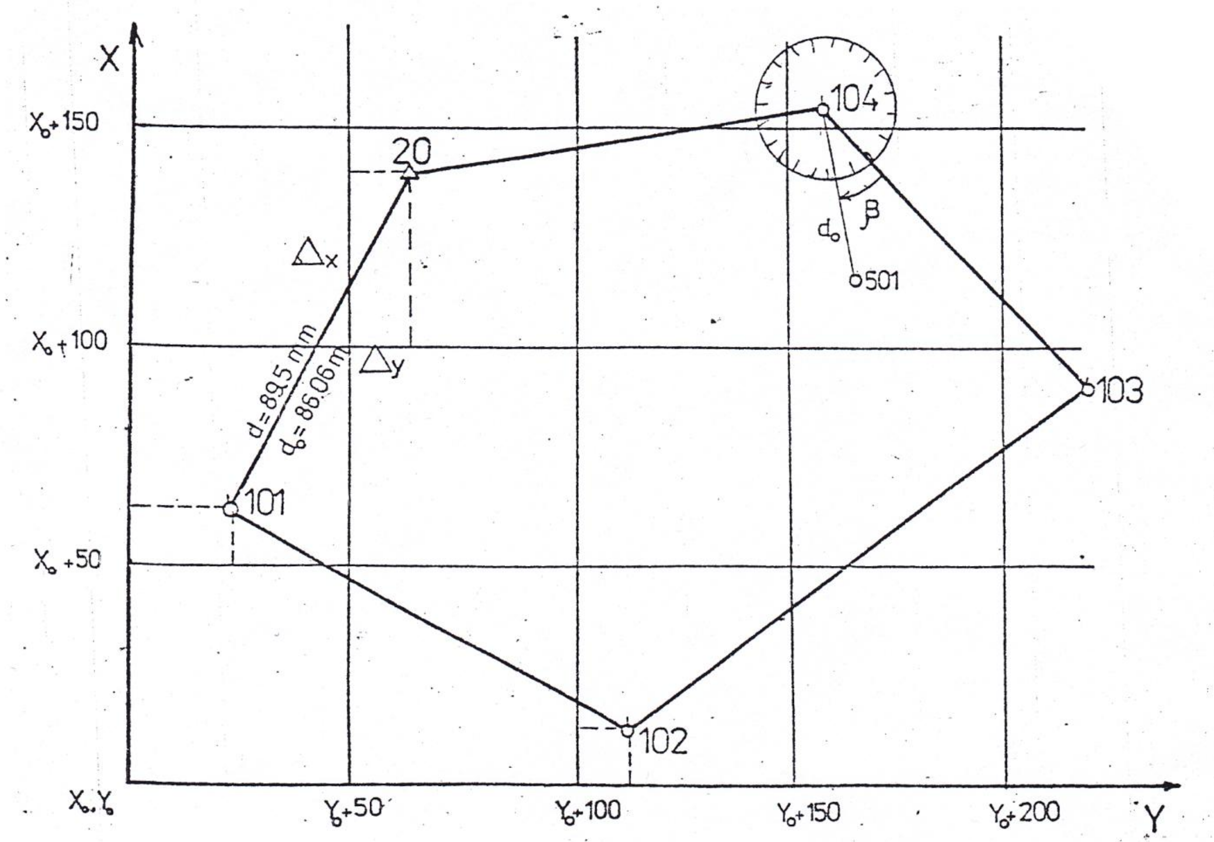


Fig.7.1

The number of the point is inscribed in the right side on horizontal.

The checking out of the plotting of points by their rectangular coordinates is made by graphic measurement of distances between points and comparing them with the ones obtained by calculation. If the differences between the two values do not outrun the plotting graphic error, the points can be regarded as correctly plotted. If contrary there has been made a plotting mistake, so will have to correct the plotting. As an example, checking out the plotted points 20 and 101 is made in the following way: the distance on the plan is measured by ruler obtaining d=89.50mm that is corresponding to a ground distance D=89.5m.The calculations have revealed for the same distance of value of d_{01}=89.06m. The difference between them, 89.5-89.06=0.44m do not outruns the maximum plotting error which is +/- 0.5 mm(for the 1:1000 scale that means 0.5m). The same way the other radiation and traverse points are cheeked out.

The plotting of radiation points by their polar coordinates is made using the protractor and the graduated ruler, from the triangulation point or traverse points, with reference to the previous side of the traverse.

For plotting the radiation point 501, the circular protector is positioned so that its center superposes over the traverse point 104 and is stiffed with a needle. The protractor is rotated until the zero graduation is superposing over the previous side 104-103, after which, using a pen, the value of the horizontal angle $\beta_{501}=36^{g}12^{c}\approx36^{g}$ is marked (fig.7.1). Along the obtained distances is measured the horizontal distance (at scale) d_0,501=30,17m(on plan d=30mm) and at its end we shall find the plane position of 501.

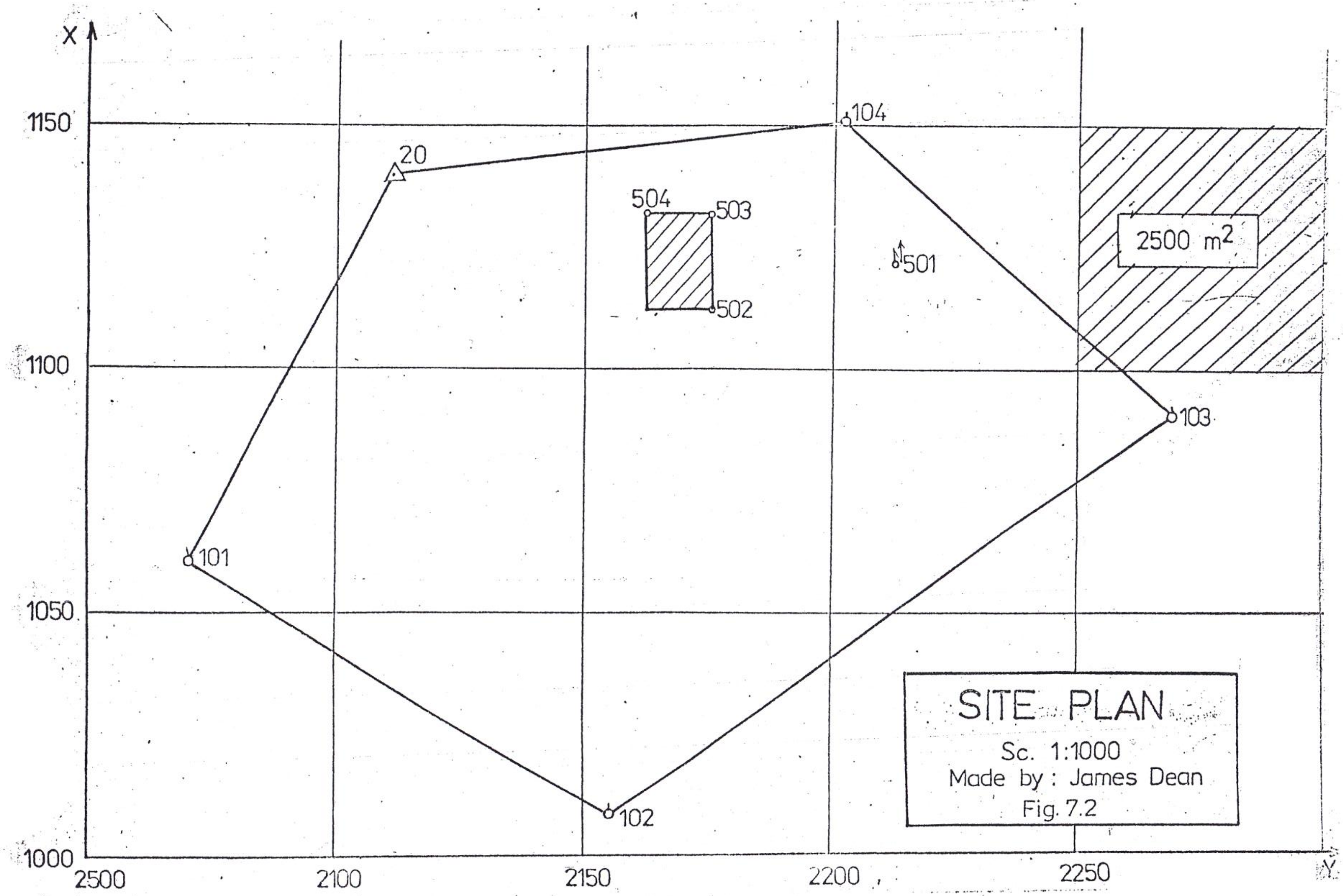
c. Connecting the points. After plotting the points of the support network(triangulation or traverse) and the characteristic points(radiation), the latter are connected obtaining in this way the shape of planimetric details. As example, by connecting points 502,503 and 504 are obtained two side of a building, the other two being obtained taking parallels to the latter.

After obtaining the details, the plan is completed with the appropriate conventional signs.

d. The cartographing of plan. In the case of site plan draw on scale paper, the original remains draw only with usual pen. The toponymy and number of points are inscribed on it, together with the title of the plan, the considered zone, the scale, the year of the surveying and the name of the operator. In the case of topographic plans made on metal mounted paper the writing is made with China ink.

The original plans are special documents that are kept in special condition at archives. For different studies and projections there are executed copies. The copying is made either directly on special trasing paper or indirectly by photographing. The multiplication of copies is made by diazo print, photographing or by printing.

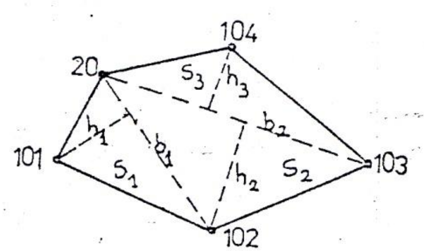
When modification or reduction of the scale of a plan is asked we can use either the method of square grid, by pantographing or by photographing. According to the described operations, the site plan(fig.7.2) has beed draw.


X
1150
1100
1050
1000
2500
2100
2150
2200
2250
Y
20
104
101
102
103
504
503
502
501
2500 m^2
SITE PLAN
Sc. 1:1000
Made by : James Dean
Fig. 7.2

2. Determination of the area inside the close circuit of the given traverse by graphic method, by two division in simple geometric shapes, together with the verification of the two division ond establishing the final result.

The inside area, being surrounded by a continuous polygonal contour, is divided in simple geometric shapes(as ex. in triangles), by connecting point 20 with points 102 and 103. As an indication the number of triangles has to be as possible small, the base of the triangles almost the same with their heights.

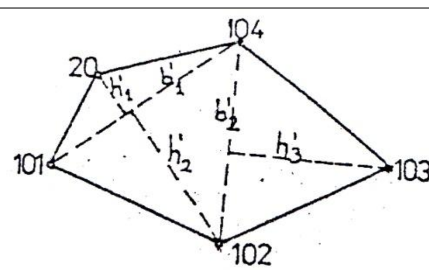
In the resulted triangles(fig.7.2), having the S_1,S_2 and S_3 surfaces, with reference to the b_1 and b_2(which is common), are taken(using a set square) the h_1,h_2 and h_3 heights. Using a ruler, graphical, the measures of the bases and heights are determined and entered in table 7.2. Depending on these measurement, the surface of the three triangles are calculated and, by their amount, the area of the considered surface is calculated(for the first division).

Index of surf.	Base (mm)	Height (mm)	Area (mm²)	Sketch
S_1	137.5	65.5	4503.125	
S_2	165.5	115.5	9226.625	
S_3	165.5	38.0	3144.500	
Area in mm² (plan) = 16874.250 Area in m² (ground) = 16874.250				

For obtaining the area in square meters, the calculated area from plan-which is in square millimeters-is multiplied with the square of the denominator of the scale.

$S_I(m^2)=S_I(mm^2)*N^2=16874.250mm^2*1000^2=16874.250m^2$

This method of calculation is comfortable especially in the case maps at scale such as 1:500, 1:2000, 1:5000 etc, where there isn't a correspondence between the plan distances and their correspondent on the ground.

Index of surf.	Base (mm)	Height (mm)	Area (mm²)	Sketch
S_1	159.0	42.0	3339.000	
S_2	159.0	91.0	7274.250	
S_3	149.5	84.0	6272.000	
Area in mm² (plan) = 16882.250 Area in m² (ground) = 16892.250				

For the control of the area determination, a new division is made in the same maner the data being entered in table 7.3. The surface that resulted from the second division is:

$S_{II}(m^2)=S_{II}(m^2)*N^2=16892.250mm^2*1000^2=16892.250m^2$

We're checking if the absolute value of the difference between the two determinations is smaller than the tolerance:

$|S_I-S_{II}|<=(1/400)*S_I$; $|-18000m^2|<42.185m^2$

If this condition is fulfilled, as a final value of the area we shall consider the arithmetic mean of the two determinations:

$S=(S_I+S_{II})/2=16883.25m^2$

3.Determination of the area using the mechanic method, using the polar planimeter, including the scale constant determination.

The mostly used type of mechanic method for calculating areas is the one using the polar planimeter.

The polar planimeter consist of two arms: the polar arm and the tracing arm(fig.7.3). The polar arm, that has a constant length, has at one of its ends weight named pole which has a needle for its plane fixing.

The tracing arm, divided into millimeters, has at is free end a point down as still, or a magnifier and at the other end a divided integrator feel together with a recording device.

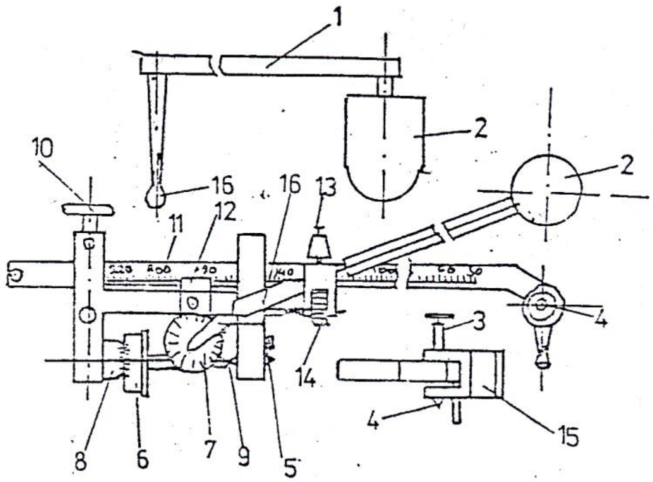


Fig.7.3

The recording device consist of a wheel divided into 100 graduations, inscribed form 10 to 10, a vernier index which has 10 graduations and a circular plate(lap counter) connected with the wheel which has as a purpose the recording of the number of laps made by the wheel during the determination.

The whole recording device can move along the tracing arm by modifying the distance to the still.

In fig.7.3 we have: 1-polar arm; 2-the pole; 3-tracing arm; 4-still; 5-recording device; 6-interrogator wheel; 7-lap counter; 8-vernier; 9-endless screw; 10-suport wheel; 11-the graduation of the tracing arm; 12-tracing arm length fixing vernier; 13- stopping screw; 14-fine motion screw; 15-driving device of the arm; 16-the joint of the two arms.

The way of working with the polar planimeter. The determination of the areas using the polar planimeter is known as planimetry . During the planimetry we have to fulfill certain conditions:

-the topographic plan will be properly mounted on a stiff and horizontal surface.
-in the starting point from the contour line the angle between the arms will be approximately right.
-the pole will be fixed so that during planimetry the angle between arms will not be too small(under 10°) or too big(over 170°).
-the tracing of the contour lines is done by hand, clockwise.

The planimerty operation is done in two ways: with the pole outside the surface which is mostly used (fig.7.4) and with the pole inside the surface.

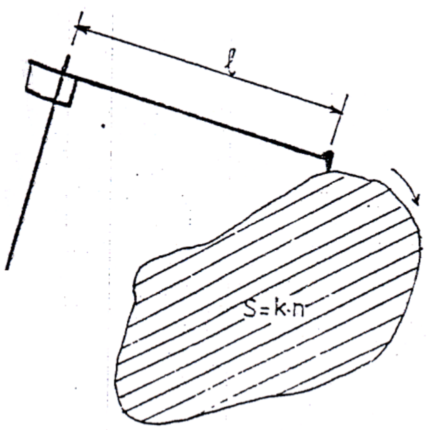


Fig.7.4

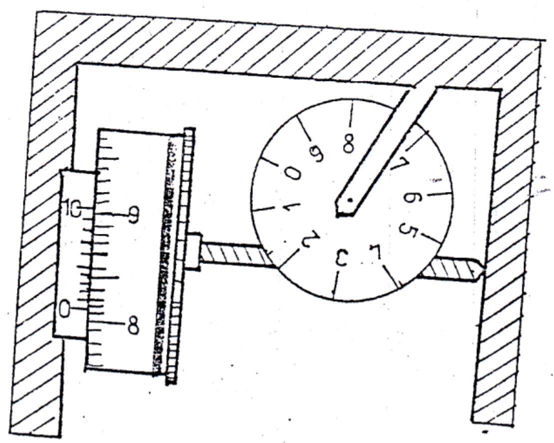


Fig.7.5

In the case of planimetric with the pole outside, the area is determinate using the following formula:

$S=K*(C_2-C_1)=K*n$

In the case of the inside pole we are using the following formula:

$S=[Q+/-(C_2-C_1)]*K=(Q+/-n)*K$

In these formulas C_1 and C_2 are the initial and final readings on planimeter, n-the generating number obtained from the difference the two readings, K-the scale constant which depends on the scale of the plan and on the length of the tracing arm, Q-the constant of the planimeter, representing the area of the base circle. The size of the two constant are given in a technical note of the instrument(table 7.4)

TABLE 7.4

Planimeter Constant Q	Usual Scales 1:N	Length of tracing arm 1	Scale constant K
23212	1:500	7.998	$2m^2$
	1:1000	10.000	$10m^2$
	1:2000	4.995	$20m^2$
	1:5000	3.995	$100m^2$
	1:10000	10.000	$1000m^2$

The reading at the planimeter is made by four numbers: the first one is red from the lap counter(fig.7.4), the second one is the number inscribed on the wheel, the third represent the number of integer graduation up to the vernier index and the fourth is red on the vernier in the point of coincidence. In fig.7.4 the reading is C_1=2816.

No matter which is the position of the pole the planimetry is made as follows:

The still of the planimeter as positioned in a district point on the contour line making at the same time the initial reading, C_1. Then, following the contour line, clockwise, till we get into the initial point where the final reading, C_2, is made. According to these two reading the area is determined using the above mentioned formulas.

When the instrument doesn't have a technical note, the K and Q constants can determinate. For the control of the measurements and the achieving a higher accuracy, the planimetry is made 3. . . 5 times. For finding the generating number the admitted interval is of five, graduations from the vernier.

$E_{max}=(n_{man}-n_{min})<=T=5$ vernier graduations

a.Determination of the scale constant. When the constants are know from the technical note of the instrument, according to the scale of the plan, the appropriate length of the tracing arm is fixed (table7.4). In the case that the technological note doesn't exist any longer, we have to determinate first the scale constant, K. The screw(13) is released and the reading device is moved to the end of the tracing arm after which the screw is locked. A know area $S_0=2500m^2$ representing a square with a 50m side from the plan is planimetred with the pole outside. The results of the three determination are entered in table 7.5.

Since the pole remains immobile, the final reading becomes initial readings for the next determination.

TABLE 7.5

NO.	Readings		Generating Number $N=C_f-C_i$	Mean Generating number	Unitary Surface S_0	$K=\frac{So}{n}$
	Initial	Final				
1	2	3	4	5	6	7
1	2573	2813	240	239.67	$2500m^2$	10.43
2	2813	3054	241			
3	3054	3292	238			

The generating number for each determination is entered in the fourth colum.

$n'=2813-2573=240$; $n''=3054-2813=241$; $n'''=3292-3054=238$

The maximum interval being E_{max}=421-238=3 units, is smaller than the tolerance. The mean generating is determined in colum 5.

$n=(n'+n''+n''')/3=239.67$

This measured is entered in the seventh column.

b.Determination of the area inside the polygonal contour. The planimerty is made with the pole outside of the closed circuit traverse area(fig.7.2). The readings and calculations are entered in table 7.6

TABLE 7.6

NO.	Readings		Generating Number $n=C_f-C_i$	Mean Generating number	K (m^2)	S=K*n (m^2)
	Initial	Final				
1	2	3	4	5	6	7

1	7453	9067	1614	1615.67	10.43	16851.46
2	9067	0682	1615			
3	0682	2300	1618			

The generating number is calculated for each determination:

n'=9067-7453=1614; n''=10682-9067=1615; n'''=2300-0682=1618

The maximum interval is smaller than the tolerance:

E_{max}=1618-1614=4<5 vernier graduations

The mean generating number is :

n=(n'+n''+n''')/3=1615.67

Depending on the latter and the scale constant, the area is calculated:

S=K*n=10.43*1615.67=16851.438m^2

4.Area determination by analytic method

This method is used for calculating the areas when the rectangular coordinates of the corner of the polygon are known. Comparing with the other method the analytic method ensures the highest accuracy and doesn't require the presence of the site or topographic plan. The general formula for the area of a n-sides polygon is:

$$2*S=\sum_{i=1}^{n} Xi *(Y_{i+1}-Y_{i-1})$$

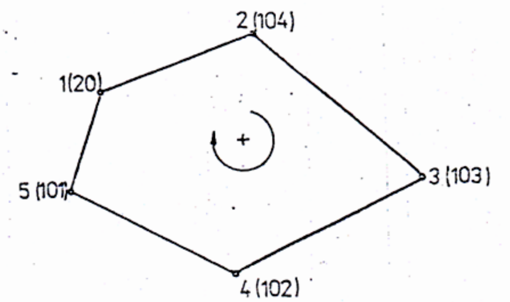
taking intro account the direct sense of inscribing points on the contour. For the given surface the double of the area is:

$$2*S=\sum_{i=1}^{5} Xi *(Y_{i+1}-Y_{i-1})=X_1*(Y_2-Y_5)+X_2*(Y_3-Y_1)+X_3*(Y_4-Y_2)+X_4*(Y_5-Y_3)+X_5*(Y_1-Y_4)=X_{20}*(Y_{104}-Y_{101})+X_{104}*(Y_{103}-Y_{20})+X_{103}*(Y_{102}-Y_{104})+X_{102}*(Y_{101}-Y_{103})+X_{101}*(Y_{20}-Y_{102})$$

The calculation are done in table 7.7 according to the given scheme, with repeating the last and the first point, in the case of an odd number of points. When we have an even number of points one of them is repeated once more, immediately under it, without mentioning the repetition of the last and the first point.

TABLE 7.7

Real point	Form. point	Rectangulare coordinates X	Rectangulare coordinates Y	Sketch
1	2	3	4	5
101	5	1061.04	2069.88	
20	1	1139.75	2111.42	
104	2	1151.12	2201.49	
103	3	1091.17	2270.08	
102	4	1009.38	2155.10	
101	5	1061.04	2069.88	
20	1	1130.75	2111.42	



The calculation is done using a minicalculator or a computer. Using a minicalculator we obtained:

$2*S=33595.717m^2$, resulting $S=2*S/2=16797.858m^2$

For controlling the calculations we'll use the formula of the double negative area:

$-2*S=+\sum_{i=1}^{n} Yi*(X_{i+1}-X_{i-1})$

or,

$-2*S=-\sum_{i=1}^{5} Yi*(X_{i+1}-X_{i-1})=-[Y_1*(X_2-X_5)+Y_2*(X_3-X_1)+Y_3*(X_4-X_2)+Y_4*(X_5-X_3)+Y_5*(X_1-X_4)]= -[Y_{20}*(X_{104}-X_{101})+Y_{104}*(X_{103}-X_{20})+Y_{103}*(X_{102}-X_{104})+Y_{102}*(X_{101}-X_{103})+Y_{101}*(X_{20}-X_{102})]$

Considering the symmetric scheme to the one of the double positive area, we shall have:

$-2*S=-33595.717m^2$, resulting $-S=-2*S/2=-16797.858m^2$

Between the two values there's no difference. Practically, there can appear small differences because of the round off of the minicalculator. In this case we shall take the mean value of the two determinations.

When we're using only the analytic method, a control can be made by planimetring the area or by graphic method. Comparing the three results of the same area obtained graphically, mechanically and analytically

$S_{gra.}=16883.25m^2$; $S_{mec}=1685.46m^2$; $S_{anal}=16797.858m^2$;

It results that concerning the accuracy, on the first place is the analytic method, then the mechanic one and at last the graphic one.

Practically the area determination is made only by one of the above mentioned methods.

Leveling Methods

For studying and projecting engineering works the largest usefulness have the maps and plans on which are represented both planimetric details and the relief forms. Leveling is completing the planimetric surveys with the forms of relief giving thus a complete image of the earth's surface that is represented on the plan.

Aplication Theme

For modernizing a street there has been made a leveling by geometric leveling from the mid-point between bench marks the elevation of which are know, combined with method of the cross section. We're asked for the determination of the elevation of point, drawing up the longitudinal profile of the streets and the transversal profiles of the streets.

Data

1. The elevation of the support bench marks, R_5 and R_8
2. The horizontal distances measured directly in the field between the points of the traverse and also from the axis of the road to each point in transversal section.
3. Readings on the road at the horizontal cross hair from each leveling station point.

OBS: The initial data (elevation) or the measured ones (horizontal distances and readings on the road) are entered in a special field (table 9.1) in columns 10, 3, 4, 5 and 6.

The application will consist of:

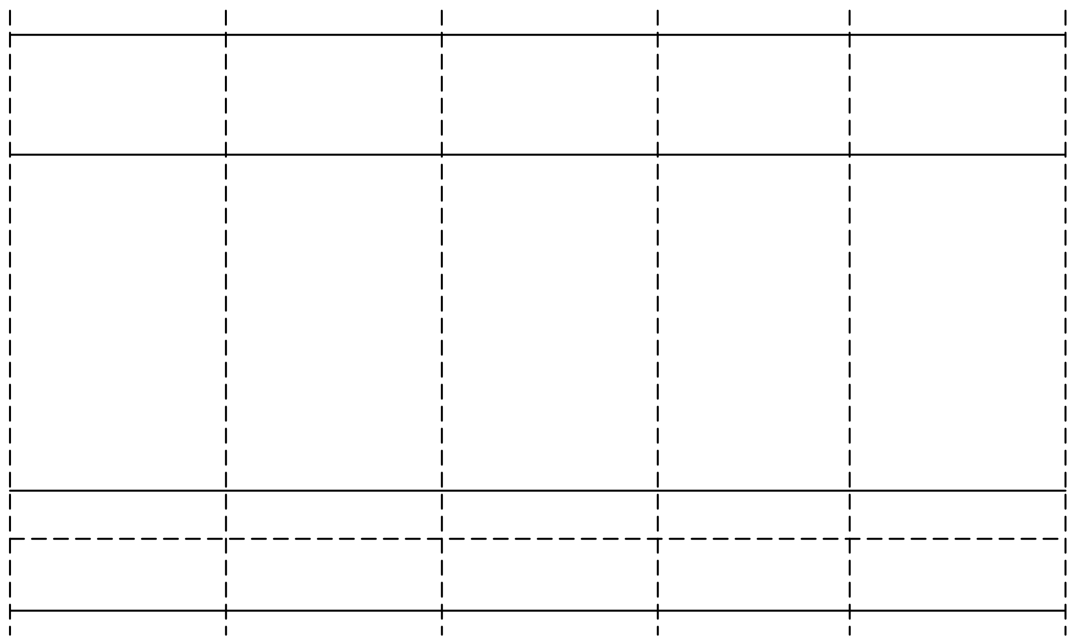
1. Determination of the difference in elevation between the traverse point.
2. Determination of the closing error on differences in elevation and their compensation.
3. Determination of the elevation of the traverse point.
4. Determination of the elevation of the points of the longitudinal profile.
5. Drawing up the longitudinal profile of the streets axis, having the length scale as 1:1000 and the elevation scale 1:100.
6. Drawing up the transversal profiles, having both the length and elevation scale as 1:100.

Table 9.1

Stat Nb.	Aimed point	Dist. (m)	Readings on road (mm)						Elevat. (m)	Mark. Nb.
1	2	3	4	5	6	7	8	9	10	11
S_1	R_5	-	1574						45.873	R_5
	108	50.000			1297					
	501	1.60		1522						
	502	1.29		1558						
	503	1.29		1710						
	504	4.28		1594						
	505	8.33		1738						
	506	8.33		1541						
	507	11.60		1512						
	508	1.65		1353						
	509	1.45		1444						
	510	1.45		1630						
	511	4.95		1537						
	512	8.48		1634						
	513	8.48		1423						
	514	11.88		1387						
S_2	108	-	1733							
	109	50.00			1421					
	515	1.69		1656						
	516	1.32		1656						
	517	1.32		1828						
	518	4.82		1677						
	519	8.33		1849						
	520	8.33		1633						
	521	11.95		1595						
	109	-	1662							
	110	50.00			1853					
	522	1.71		1831						
	523	1.42		1876						
	524	1.42		2031						
	525	4.95		1873						
	526	8.46		2023						
	527	8.46		1849						
	528	12.50		1811						
S_4	110	-	1726							
	111	50.00			1514					
S_5	111	-	1596							
	R_8	72.75								

Solution

For the leveling of a portion of a street, having as a purpose a project of expanding and modernizing this streets, a geometric mid-point leveling traverse has been made, R_5 – 108 – 110 – 111 – R_8, having as a support two bench marks R_5 and R_8 whose elevation are know belonging to the leveling network, combined with the method of the cross section (fig. 9.1)



1. Determination of the difference in elevation between the traverse points

In each station point of geometric mid-point leveling the difference in elevation between the traverse points are obtained marking difference between the rear and forward readings. The difference in elevation will be:

$$\Delta_{Z1} = L_o^{R5} - L_o^{108} = 1572 – 1297 = + 275 \text{ mm}$$

$$\Delta_{Z2} = L_o^{108'} - L_o^{109} = 1733 - 1421 = + 312 \text{ mm}$$

$$\Delta_{Z3} = L_o^{109'} - L_o^{110} = 1662 - 1853 = - 191 \text{ mm}$$

$$\Delta_{Z4} = L_o^{110'} - L_o^{111} = 1726 - 1514 = + 212 \text{ mm}$$

$$\Delta_{Z5} = L_0^{111'} - L_o^{R8} = 1596 - 1221 = + 375 \text{ mm}$$

2. **Determination of the closing error on differences in elevation is made:**

$$\sum_{I=1}^{5} \Delta_{Zi} = 275 + 312 - 191 + 212 + 375 = 983 \text{ mm}$$

and also the differences between the elevation of the support bench marks that are entered in the 10-th column of the table 9.1.

$$\Delta z_{R5-R8} = Z_{R8} - Z_{R5} = 46.847 - 45.873 = 0.974 \text{ m} = 974 \text{ mm}$$

Theoretically, the algebraic amount of the differences in elevation measured on the ground between the support bench marks should de equal with the known differences in elevation resulting from the differences of the elevation of the bench marks. Practically, because of the measuring and instrumental errors this conditions is not fulfiled. The algebraic differences, positive or negative, between the two values is know as closing error on differences in elevation, give as:

$$F_Z = \sum_{i=1}^{5} \Delta Z_i - \Delta Z_{R5-R8} = 983 \text{ mm} - 974 \text{ mm} = +9 \text{ mm}$$

The numerical value of the error has to be smaller than the tolerance which is given by the technical instruction for each type of leveling. In the case of the 4-th order leveling in on site buildings the tolerance is:

$$T_Z = +/- \ 20 \text{ mm} * \sqrt{D_{km}} = +/- \ \sqrt{0.27275} = +/- \ 10.44 \approx 11 \text{ mm}$$

where D = the total length of the traverse

$$D = \sum_{i=1}^{5} d_i = 50 + 50 + 50 + 50 + 72.75 = 272.75 \text{ m}$$

the above mentioned condition being fulfield,

$$F_z <= T_Z \text{ , i.e. } 9 \text{ mm} < 11 \text{ mm}$$

Other on, we can compensate the differences in elevation, operation that insist in aplying corrections to the differences in elevation, corrections that are proportional to the distances between points.

The correction will be equal in magnitude with the error but it will have an opposite sign to the latter: $C_Z = - f_Z = - 9$ mm

Relying on the total correction we can determinate the unitary correction per meter

$$C_Z = \frac{C_z\ (\text{mm})}{D\ (\text{m})} = -\frac{9\ \text{mm}}{272.75\ \text{m}} = -0.033\ \text{mm/m}$$

Then, the corresponding corrections for each differences in elevation is calculated:

$$\Delta_{Z1} = C_2^u * d_1 = -0.033 * 50 \quad = -1.65 \approx -2\ \text{mm}$$

$$\Delta_{Z2} = C_2^u * d_2 = -0.033 * 50 \quad = -1.65 \approx -2\ \text{mm}$$

$$\Delta_{Z3} = C_2^u * d_3 = -0.033 * 50 \quad = -1.65 \approx -2\ \text{mm}$$

$$\Delta_{Z4} = C_2^u * d_4 = -0.033 * 50 \quad = -1.65 \approx -1\ \text{mm}$$

$$\Delta_{Z5} = C_2^u * d_5 = -0.033 * 72.75 = -2.40 \approx -2\ \text{mm}$$

As a verification, the amount of partial correction are entered in the 7-th column of the table 9.2 under the differences in elevation. The compensated references in elevation are obtained by adding algebraically the initial reference in elevation with the partial correction, resulting:

$$\Delta_{Z1}^C = \Delta_{Z1} + C\Delta_{Z1} = +275 - 2 = +273\ \text{mm}$$

$$\Delta_{Z2}^C = \Delta_{Z2} + C\Delta_{Z2} = +312 - 2 = +310\ \text{mm}$$

$$\Delta_{Z3}^C = \Delta_{Z3} + C\Delta_{Z3} = -191 - 2 = -193\ \text{mm}$$

$$\Delta_{Z4}^C = \Delta_{Z4} + C\Delta_{Z4} = +212 - 1 = +211\ \text{mm}$$

$$\Delta_{Z5}^C = \Delta_{Z5} + C\Delta_{Z5} = +375 - 2 = +373\ \text{mm}$$

As a verification of the compensation we have to check out the equality:

$$\sum_{i=1}^{5} \Delta_{Zi}^C = \Delta z_{R5\text{-}R8} = 974\ \text{mm}$$

The compensated values are entered in the 8-th column on the respective line.

3. Determination of the elevations of the traverse points

The elevation of the traverse points are calculated using the elevation of the initial bench mark $Z_{R,}$ to which we added algebraically, succesivlly and cumulated the compensated difference in elevation:

$$Z_{108} = Z_{R5} + \Delta z_1^{C} = 45.843 + 0.273 = 46.146 \text{ m}$$

$$Z_{109} = Z_{108} + \Delta_{Z2}^{C} = 46.146 + 0.310 = 46.456 \text{ m}$$

$$Z_{110} = Z_{109} + \Delta_{Z3}^{C} = 46.456 - 0.193 = 46.263 \text{ m}$$

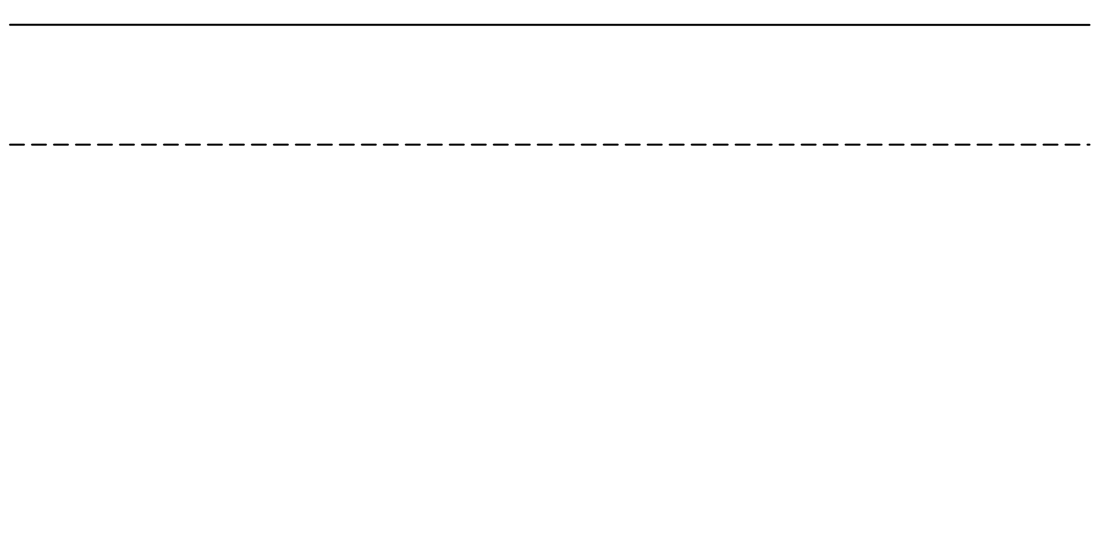
$$Z_{111} = Z_{110} + \Delta_{Z4}^{C} = 46.263 + 0.211 = 46.474 \text{ m}$$

$$Z_{112} = Z_{111} + \Delta_{Z5}^{C} = 46.474 + 0.373 = 46.846 \text{ m}$$

The traverse being compensated, the elevation of the final bench mark thus calculated should have been equal with its initial values. The values of the elevation of the traverse points are entered in the 10-th column.

4 . Determination of the elevation of the points of the longitudinal profile

The points from the transversal profiles are characteristic points, that is, points where we have a modification of the slope of the round. In the case of leveling a street, the characteristic point are indicated in fig. 9.2



The elevation of points from the profile are calculated with reference to the elevation of the plane of sight according to which there have been made measurements.

As an example, for points 501, 502, …, 514 measured from the station point, S_1, the calculations of the elevation is made as below:

- the elevation of the plane of sight of the level from the station point S_1 is calculated using the known elevation Z_{R5} and Z_{108} and the readings on the rods in these points.

$$Z_{pv}^{(1)} = Z_{R5} + L_o^{R5} = 45.873 + 1.572 = 47.445 \text{ m}$$

and for control:

$$Z_{pv}^{(1)} = Z_{108} + L_o^{108} = 46.146 + 1.297 = 47.443 \text{ m}$$

The differences between the two values which is of 2 mm is because of the compensation and in the following calculation who shall consider the mean value which is entered on the line of S_1 in the 9-th column.

- the elevation of the points from the profiles are calculated by substracting from the elevation of the plane profiles are calculated by substracting from the elevation of the sight the reading on the road in each point, taken from the 5-th column, obtaining:

$$Z_{501} = Z_{pv}^{(1)} - L_o^{501} = 47.444 - 1.552 = 45.692 \text{ m}$$

$$Z_{502} = Z_{pv}^{(1)} - L_o^{502} = 47.444 - 1.558 = 45.886 \text{ m}$$

– –

$$Z_{514} = Z_{pv}^{(1)} - L_o^{514} = 47.444 - 1.387 = 46.057 \text{ m}$$

The elevation are entered in the 10-th column on the line of each point. This way are calculated the elevation of all the other points from the profiles, with reference to the elevation of the plane of sight if the station from which those points have been aimed.

5. Drawing up the longitudinal profile of the street axis, having the length scale as 1:1000 and the elevation scale as 1:100.

The model of making profile (section) is the same way as in application nb. 2, the only difference being in the way of obtaining the accessory elements for making the profile. In the case, the horizontal distances and the elevation have been obtain by ground measurement and office calculation.

On the length scale are positioned the points from the axis of the street given as 504, 511, 518, …, with the ground completed with the requested elements. On the elevation scale are described in elevation of integer values 45, 46 and 47 m at the 1:100 scale (1mm – 1m). Depending on the latters, perpendiculars are taken from the points of the length scale up to the respective level.

The ends of the perpendiculars are connected obtaining the longitudinal profile of the axis of the actual street that is going to the modernized. As an example the rate of grade for the 504-511 alignament will be:

$$P\% \;=\; 100 * \frac{Z_{511} - Z_{504}}{d_{504-511}} = 100 * \frac{45.907 - 45.850}{50} = +\,0.1\%$$

6. Drawing up transversal profiles having both length and elevation scales as 1:100

The way of making the transversal profile from point R_5 is the same as in the case of the longitudinal profile, the only differences being thet we have both scales as equal (1:100).

For the transversal profile R_5, the first point will be 501. For positioning the point on the length scale, since distances have been measured from the traverse axis to each point, we shall use the lengths:

$$d_{501\text{-}502} = d_{501\text{-}503} = d_{R5\text{-}501} + d_{R5\text{-}502} = 1.60 + 1.29 = 2.89 \text{ m}$$

$$d_{502\text{-}504} = d_{503\text{-}504} = d_{R5\text{-}504} - d_{R5\text{-}502} = 4.84 - 1.29 = 3.53 \text{ m}$$

$$d_{504\text{-}505} = d_{504\text{-}506} = d_{R5\text{-}505} - d_{R5\text{-}504} = 8.33 - 4.82 = 3.51 \text{ m}$$

$$d_{505\text{-}507} = d_{506\text{-}507} = d_{R5\text{-}507} + d_{R5\text{-}505} = 11.60 - 8.33 = 3.27 \text{ m}$$

The transversal profile R_3 is presented in fig 9.4. on the last line of the cartouche is inscribed the rate of grade and it is calculated its value. The same procedure for the other profiles from the traverse point 111 and R_8 haven't been presented anymore.

Tabel 9.2

Stat. Nb.	Aimed point	Dist. (m)	Readings on rod (mm)			Diffr. in elev.		Elev. of the plane Of sight	Elevat.	Mark Nb.
						Temp.	Comp.			
1	2	3	4	5	6	7	8	9	10	11
	R_5	-	1572			+275	+ 273	47.444	45.873	R_5
	108	50.00			1297	-2			46.146	108
	501	1.60		1522					45.892	501
	502	1.29		1558					45.886	502
	503	1.29		1710					45.734	503
	504	4.82		1594					45.850	504
S_1	505	8.33		1738					45.706	505
	506	8.33		1541					45.903	506
	507	11.60		1512					45.932	507
	508	1.65		1353					46.091	508

	509	1.45		1444					46.000	509
	510	1.45		1630					45.814	510
	511	4.95		1537					45.907	511
	512	8.48		1634					45.810	512
	513	8.48		1423					46.021	513
	514	11.88		1387					46.057	514
S_2	108	-	1733			+ 312	+310	47.878	46.146	108
	109	50.00			1421	- 2			46.456	109
	515	1.69		1656					46.222	515
	516	1.32		1655					46.233	516
	517	1.32		1828					46.050	517
	518	4.82		1677					46.201	518
	519	8.33		1849					46.029	519
	520	8.33		1633					46.245	520
	521	11.95		1595					46.283	521
S_3	109	-	1662			- 191	-193	48.116	46.456	109
	110	50.00			1853	- 2			46.263	110
	522	1.71		1831					46.285	522
	523	1.42		1876					46.240	523
	524	1.42		2031					46.085	524
	525	4.95		1873					46.243	525
	526	8.46		2023					46.093	526
	527	8.46		1849					46.267	527
	528	12.05		1811					46.305	528
S_4	110	-	1726			+212	+211	47.989	46.263	110
	111	50.00			1514	-1			46.474	111
S_5	111	-	1596			+ 375	+ 373	48.069	46.474	111
	R_8	72.75			1822	-2			47.847	R_8

Application no. 10

SETTING OUT A BUILDING ON THE GROUND

SETTING OUT A BUILDING ON THE GROUND

Setting out constructruction projects (building , ways of communication ,art works ,water constructional works) implies the solving of the inverse topographic problem, by field and office work during which we have to fulfil several conditions :

- the topographic preparing of the project for its applying on the ground (the office stage),

-setting out on the ground the main and secondary axis and the characteristical points of the projects building and completing of the setting process until the work on that building is finished .

In setting out the project the field works has ti ensure the abiding of the projected shape and dimensions of the building , including the mutual position of the latter with some other surrounding constructions. This objective can be reached only if the tophographic works are done with the appropriate precision .

For different kinds of construction , there have been elaborated standards that gives one the imposed accuracy of the setting out works (STAS 9824/0….7-74).

Applications theme

For laying out a building it is necessary the setting out on the ground of the plane and vertical position of the building project.

A topographic preparatory stage of the project will be done for setting on the ground the plane position by using the following methods : the methods of the polar coordinates the method of the angular intersection and the method of the linear intersection.

Also there will be indicated the wey of working in applying each method including the setting out construction elevation zero using the method of the plane of sight by which the vertical position of the building is to be made.

Data

1. **The topographic plan with the location of the building (fig .10.1)**
2. **The coordinates of the support network prints 101 and 102 which already exist from the previous topographic .**
3. **Coordinates of the two of the corners of the projected building C1 and C2 read graphicly with reference to the system of rectangulare axis in which the topographic plan has been made .**
4. **The elevation of the height marks , RN3, that ensures the transmition and control of the projected elevation and the reading on the rod in that point .**
5. **The zero elevation of the projected building .**

OBSERVATION :

The coordinates of the support network points the onesof the projected points and of the height mark are entered in table 10.1

TABLE 10.1

Point nb	Absolute coordinates		
	X	Y	Z
1	2	3	4
101	1061.05	2069.87	—
102	1009.39	2155.11	—
C1	1070	2120	63.46
C2	1055. 00	2145	63.46
RN3	1040.12	2120.27	62.935

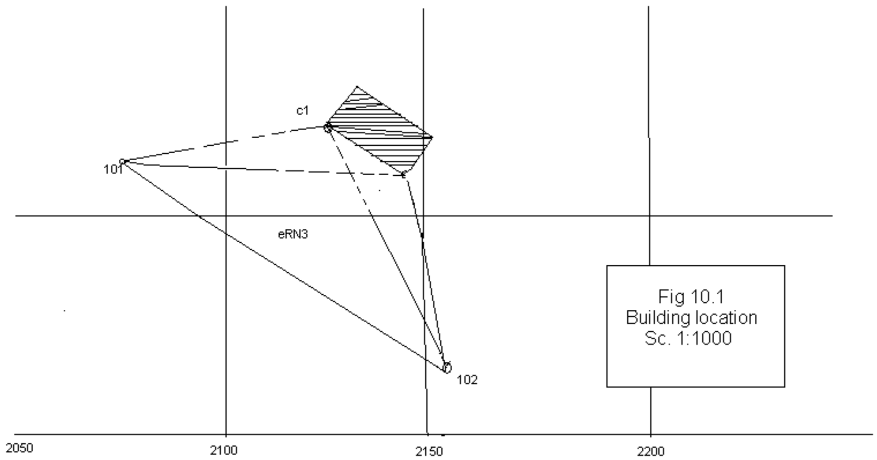


Fig 10.1
Building location
Sc. 1:1000

The application will include :

1. The topographic of the project for setting out : the determination of the topographic elements for setting out the plane position of the corners of the plane location of the corners of the building angles and horizontal distances.

2. Setting out the ground the elements that ensune the plane position of the corners of the building using the following methods : the method of the polar coordinates the method of angular intersection and the method of linear intersection .

3. The way of carryng out the operation of applying the zero elevation of the building using the method of the plane of sight

SOLUTION

After finishing the projet of the building at the same tine with she site organizing we can start the topographic operations of setting out the plane of the building on the ground .

The topographic preparing of the setting out project has to ensune :

- **Establising the points of the building that is to be sett out the accuracy of this operation and the terms of the setting out .**
- **Establising the points of the support network used foe setting out depending on which this operation will be accomplished .**
- **Chosing the setting out methods and instruments for an imposed precision .**
- **Determination according to the project of all the setting out topographic elements (horizontal and inclined distances , horizontals and vertical angles ,execution elevations) which are necessary .**
- **Organizing and management of the setting nout works ; the order of setting out ,working teams ,instruments accuracy and setting working graphs .**

For making the setting out projects ,the desingner will take account of the fact that all projected points should be determinated in the same system of coordinates as the one of the topographic plan .

For large works ,with a great number of points all the characteristical points of the projected points buildings are determinated by numerical calculations .

For small and isolated constructions , the coordinates of the points of the projected building will be , determinated graphicly , on the plan exactly as in the case of this building .

Relying on the coordinates of the setting out support network 101 and 102 ,which already exist and on the coordinates of the points of the building , during the office work are calculated the setting out topographic elements .

The setting out field operations , made in the following order

1. **The topographic preparing of the project for setting out the determination of the topographic elements for setting out the plane position of the corners of the building angles and horizontals distances**

The setting out elements of the building are depending on the chosen methond in the plan of the plan position of points .This in case of the method of polar coordinates , during the office work stage are calculated the setting out elements the ($\beta 1$ and$\beta 2$)and the control ones

(β’1and β).are calculated as also the horizontal distances (d◦1and d◦2)and the control ones (d’◦1andd’◦2).

According to fig 10.2 we can notice that the horizontal angles can obtained as differences of the grid bearings. That is way using the numeric and graphic coordinates of points , first will determinate the grid bearings and the same time with their control the values of the horizontals distances .

As an example for the bearing 101 –C1,first we all calculate the relative coordinates:

$\Delta x101\text{-}C1=XC1\text{-}X101=1070.00\text{-}1061.05=+3.95m$

$\Delta y101\text{-}C1=YC1\text{-}Y101=2120.00\text{-}2069.87=+50.13m$

Since both relative coordinates are positive,that means that the grid bearing will be in the first quadrant (see application nb.1).The calculation angle is:

$\beta 1=arctg(|\Delta y|/|\Delta x|)=arctg(50.13/8.95)=59°75'26''$

After which ,the grid bearing of 101-C1 will be:

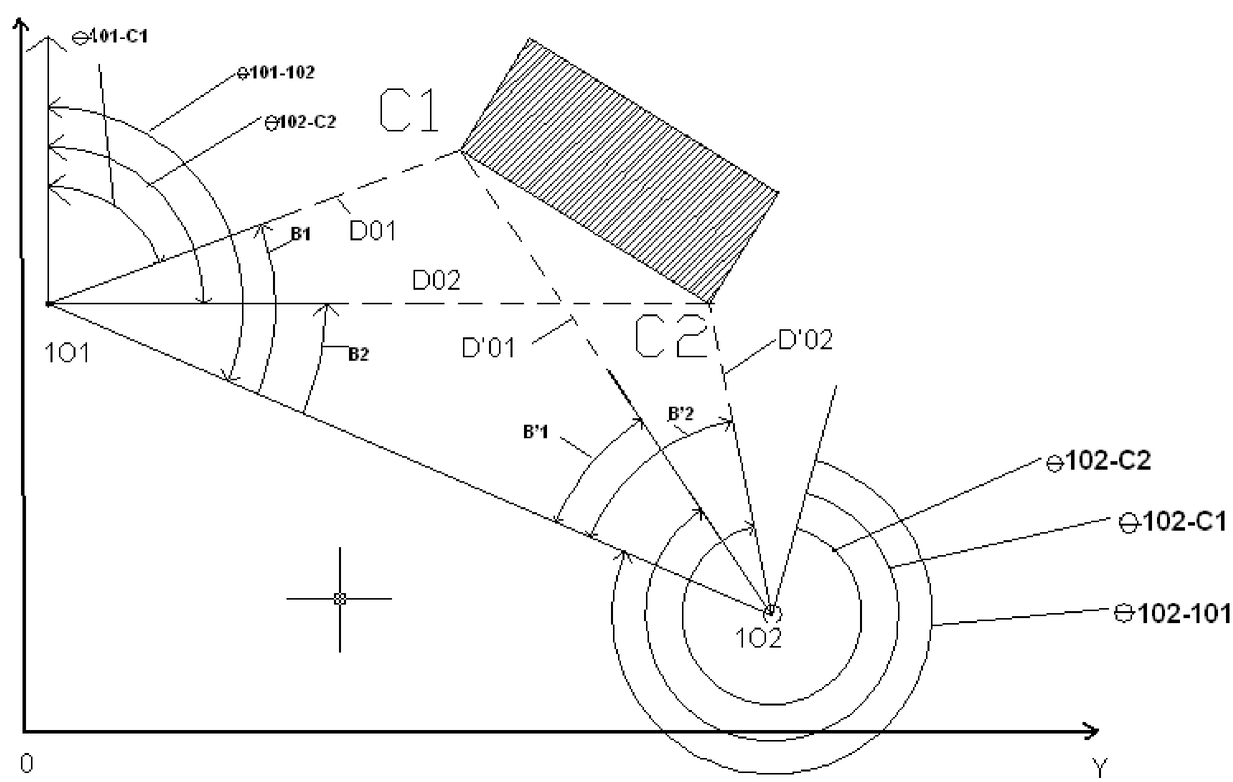


FIG 10 .2

θ **101-C1=β1=88°75'26"**

The horizontal distance between the two points will be:

$$do1 = \frac{\Delta x101 - c}{\cos\theta 101 - c1} = \frac{8.95}{\cos 85°75'26''} = 50.92$$

and for control.

$$do1 = \frac{\Delta x101 - c}{\sin\,\theta 101 - c1} = \frac{50.13}{\sin\,85°75'26''} = 50.92$$

The equality of the values of the same distance is a control of the calculation.The same way are determinated the grid bearings and the horizontal distance directly in a special table (table 10.2).For each grid bearing the plane rectangular coordinates of the initial and final point are entered in column 3 and 4 under these values being entered also the relative coordinates with their sign.The quadrant is established depending on the signs (column5) and in column 6 is made an approximative sketch including the bearing,the calculation angle and the grid bearing in column 8 are entered the natural of values of the trigonometrical functions sinus and cosinus of the grid bearings with the respective signs and the resulting distance between the two control formulas.This way the setting out topographic elements of the corners of the building will be:

-for corners c1 sett out from point 101:

{ β1=θ**101-c1=134°68'68''-86°75'26''=45°93'42''**

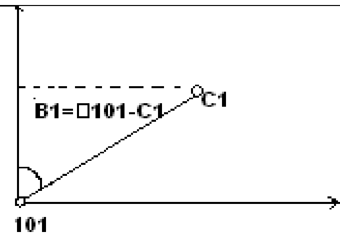
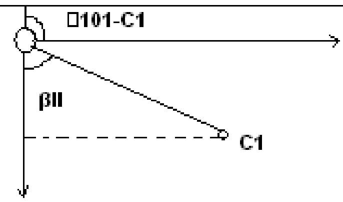
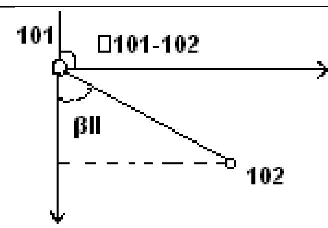
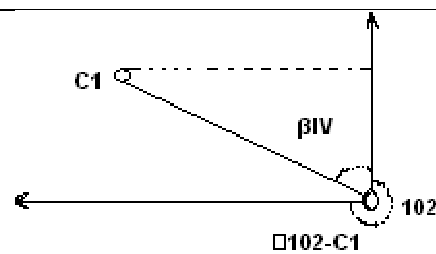
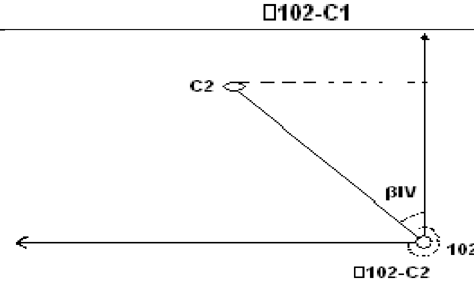
do1=50.92m

– for corners c2,sett out from point 101:

β=θ**101-102-**θ**101-c2=134°63'65''-105°11'55''=29°57'13''**

do2=73.37m

TABEL 10.2

Grid bear	Point nb	Absolute coord x	Absolute coord Y	Quad	Sketch	Tg β	Cos θ Sin θ
1	2	3	4	5	6	7	8
θ **101-c1**	C1	1070.00	2120.00	I		5.601117	0.175757
	101	1061.05	2069.87			88°75’26’’	0.984434
	Δ	+8.95	+50.13			88°75’26’’	50 , $\frac{92}{92}$
θ **101-c2**	C2	1055.00	2145.00	II		12.418181	-0.080268
	101	1061.05	2069.87			94°88’45’’	0.996773
	Δ	-6.05	+75.13			105°11’55’’	75 ,$\frac{37}{37}$
θ **101-102**	102	1009.39	2155.11	II		1.650019	-0.518298
	101	1061.05	2069.87			65°81’32’’	0.855200
	Δ	-51.66	-85.24			134°68’68’’	99 $\frac{67}{67}$
θ **102-c1**	C1	1070.00	2120.00	IV		0.579277	0.865302
	102	1009.39	2155.11			88°42’53’’	-0.501351
	Δ	+60.61	-35.11			366°57’47’’	70 $\frac{04}{04}$
θ **102-c2**	C2	1055.00	2145.00	IV		0.221682	0.976309
	102	1009.39	2155.11			18°88’69’’	-0.216409
	Δ	+45.61	-10.11			386°11’31’’	46 $\frac{72}{72}$

The control of the sett out on the ground of the projected points is made by double line setting,using independent setting out elements.So the control setting out elements will be:

-for corners c1 sett out from point 102

$\beta 1=\theta 102\text{-}c1\text{-}\theta 102\text{-}101=366°57'47''\text{-}334°68'68''=31°88'79''$

do1=70.04m

As an observation,the grid bearing of 102-101 is no longer made by its coordinates but as reserve grid bearing 101-102 so that,

$\theta 102\text{-}101=\theta 101\text{-}102+200°=134°68'68''+200°=334°68'68''$

-for corners c2 sett out from from point 102

$\beta 2=\theta 102\text{-}c1\text{-}\theta 102\text{-}101=386°11'31''\text{-}334°68'68''=51°42'63''$

do2=46.72m

The setting out and control elements for the two corners of the buildings are entered in table 10.3.

TABLE 10.3

POINT NB	SETTING OUT ELEMENTS FROM POINT `101		CONTROL SETTING OUT ELEMENTS FROM POINT 102	
	β	do	β'	d'o
1	2	3	4	5
C2	45 93'42''	50.92 m	31 88'79''	70.04 m
C3	29 57'13''	75.37 m	51 42'63''	46.72 m

For eliminating any suspicion on appering errors , the control of the setting out elements is also made by graphic measurements on the plan using the protractor and ruler .

Using the setting out elements we can draw up the setting out general plan on which are inscribed the numeralvalues of the projected building (fig.10.3).

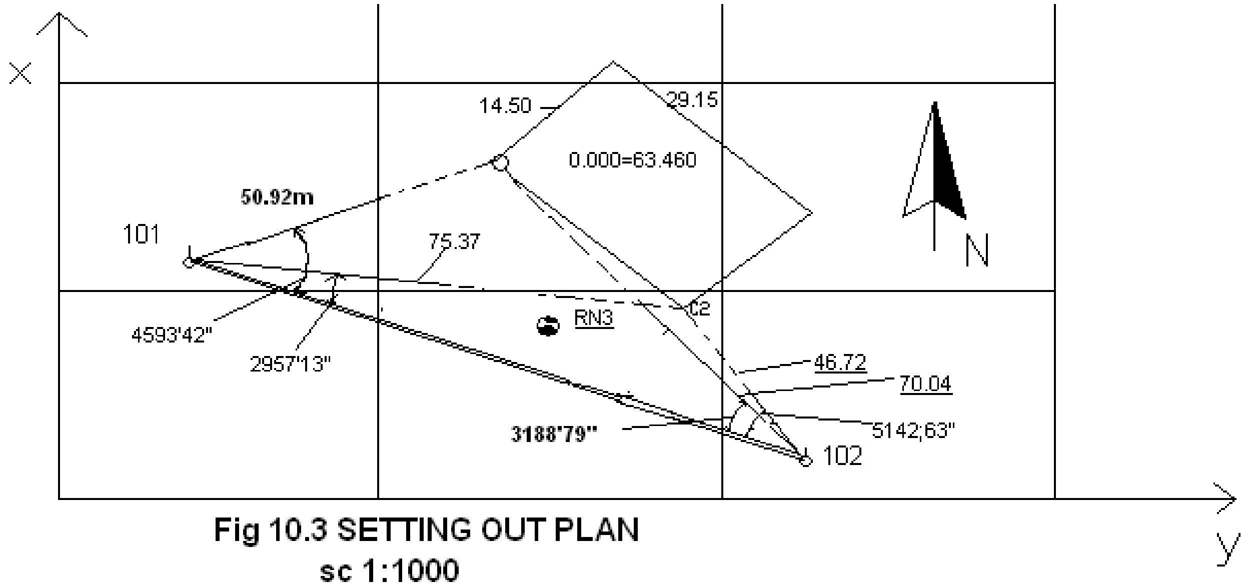
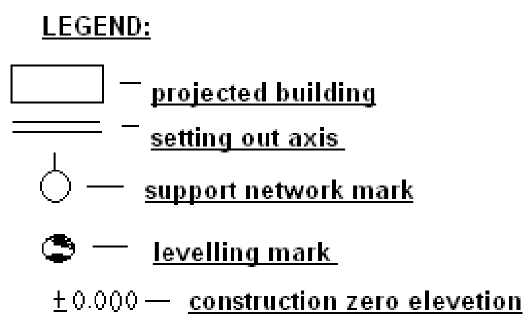


Fig 10.3 SETTING OUT PLAN
sc 1:1000



2 Position of the corners of the building using the following methods : the method of the coordinates , the methods of angular intersection , and the method of linear intersection

After the topographic preparing of the setting out project we should proced stage number two which consist of the setting out on the ground the axix and the characteristic points of the projected building for laying out the letter .

Chosing the setting out method depeds on the required precision on the type of building , the relief , the density of the points of the support network and also on the available instruments.

- **<u>The method of polar coordinates .</u> Consist in setting out on the ground the plan position of the projected building C1 and C2 from points 101 and 102 of the support network using as setting out elements :the horizontal angles , B, and the horizontal distances ,d◦.This method is oftenly used in place fields or almost plane fields that enable the neasurenments process to be done in optimal conditions (anglesand especially horizontal distances)**

The operation of setting out polar C1 is made as bellow (fig 10.4):

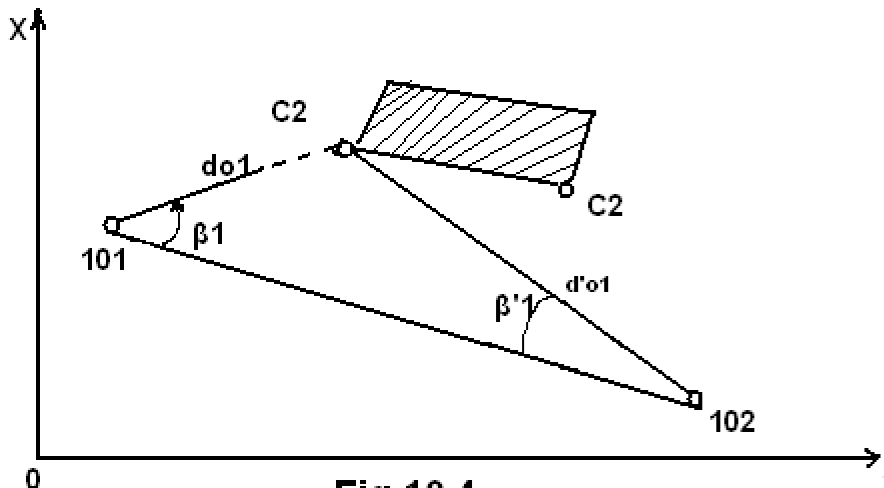


Fig 10.4

– the theodolite is sett up into the know point 101 being also centerd .

– the signal from the known point 102 is aimed and the reading on the horizontal circle is made, C1=121 35' 42''.

–the aliedade is rotated inversely until on the horizontal circle we will have a recording reading as c2=c1-β1=121 35'42''-45 39'42''= **and the bearing is marked with a ranging pole .**

–on the obtained bearing we all lay out the horizontal distance d◦1=50.92 m ,starting from point 101 obtaining this point C1which is marked by a pegg.

In the same way using the setting out elements given as β2=29 57'13'' and d◦2=75.37m we can fine point C2 .

The control of the setting out points C1 and C2 is made by setting out from the known point 102 with reference to the known direction 102 101 using the setting out elements β'1 and d'◦1 and β'2 and d'◦2.

The supplementary control is made by comparing the distance between C1and C2 with distances measured on the ground after setting . The differents between the two values doesn't have to outrun 1…2 cm.

The two other corners of the building are obtained by measuring on the perpendicular taken from C1 and C2 from thecC1 –C2 side the with l = 14.50 m

- <u>The method of the angular intersection</u> It consist in setting out on the ground the plane position of the points C1 and C2 using the points of the support network ,101 and 102 by applying the horizontal setting out angles ,β, with reference to the determinated distances .This method is used in the case of hard relief terrains where the direct measuring of distances is very difficult or even impossible.

The operation of setting out point C1 is made as follows (fig 10.5)

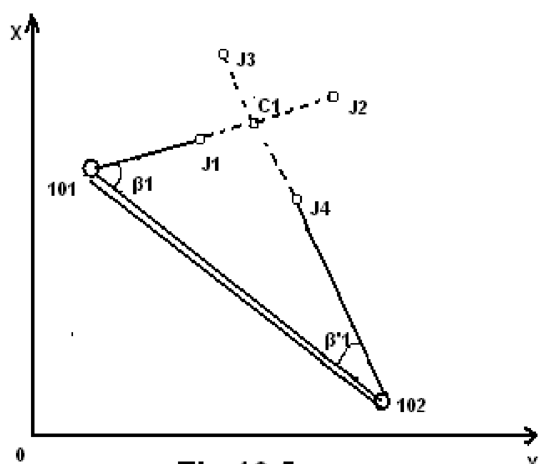


Fig 10.5

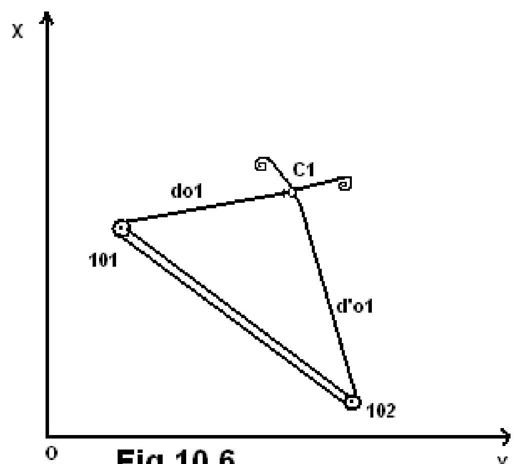


Fig 10.6

 the theodolite is sett up in the known point 101 and it is centered .

 the signal from the known point 102 is aimed and the reading on the horizontal circle is made as an example c1 =221 62'15'' .

— the alidade is rotated inversely according to the sketch until on the horizontal circle we all have a reading c2=c1-β1=221 62'15''-45 53'42''=176 08'73'' and this direction is marked with the ranging poles J1 and J2 .

—the theodolite is moved into the known point 102 centered and point 101 is aimed making the reading on the horizontal circles , as an example C3 =131 45'62''.

— the alidade is rotated clockwise(directly) until we shall record the reading C4=C3-β'1=131 45'62'' – 29 57'13''=161 02'75'' and the bearing is marked with the ranging poles J3and J4.

— at the intersection of two measuring tapes wires sett along the two directions J1-J2 and J3 –J4 we obtained the position of point C1 which will be marked with a peg .

The same way using the setting out angles β2 and β'2,point c2 is sett out.

A more precise setting out can be achieved by using simultaneously two theodolites in each point 101and 102 using the engles β1 and β'1 for C1 and β2 and β'2 for C2 obtaining thus directly the two points.

The plane position of the points c1 and c2 or the projected building using the points of the support network 101 and 102 by applying the horizont setting out distances d0 and d0.

The method ensures a high setting out precision in plane field but is limited by the fact that the setting out distances have to be smaller than the lenght of the steel measuring tape.

The operation of setting out point c1 is made as bellow (fig.10) known point 101 and it is layed up to the division corresponding to the horizontal setting out distance,do1=50.92m.

-the zero graduation of the second steel measuring tape is fixed into the known point 102 and it is layed up to the graduation corresponding to the horizontal setting out distance do1=73.37m.

-the ends of the tape are brought near untill the superpose,obtaining thus the wanted point c1.The some way,using the distances do2 is sett out point c2.

3.<u>The way of carrying out the operation of applying the zero evaluation of the building using the method of the plane sight.</u>

The elevations of the plans or isolated points of the project are being sett with reference to an arbitrary level,being positive over this level and negative under it.The value of the arbitrary level which is corresponding to a certain elevation is mentioned in the projected.

In the case of a building the arbitrary level is known construction elevation zero and represents the elevation of the finished floor of the ground story.In this case the construction elevation zero,according to the project is +/- 0.000=63.460m and it is inscribed on the setting out plan.

The construction elevation zero is sett out on the ground being geometric levelling or by trigonometric levelling depending on the relief.The construction zero elevation is sett out a few meters away from the location of the building on a peg pillow or on an existing structure.From this point the zero elevation is transmitted by using the levels based on the principle of the communicating vessels.

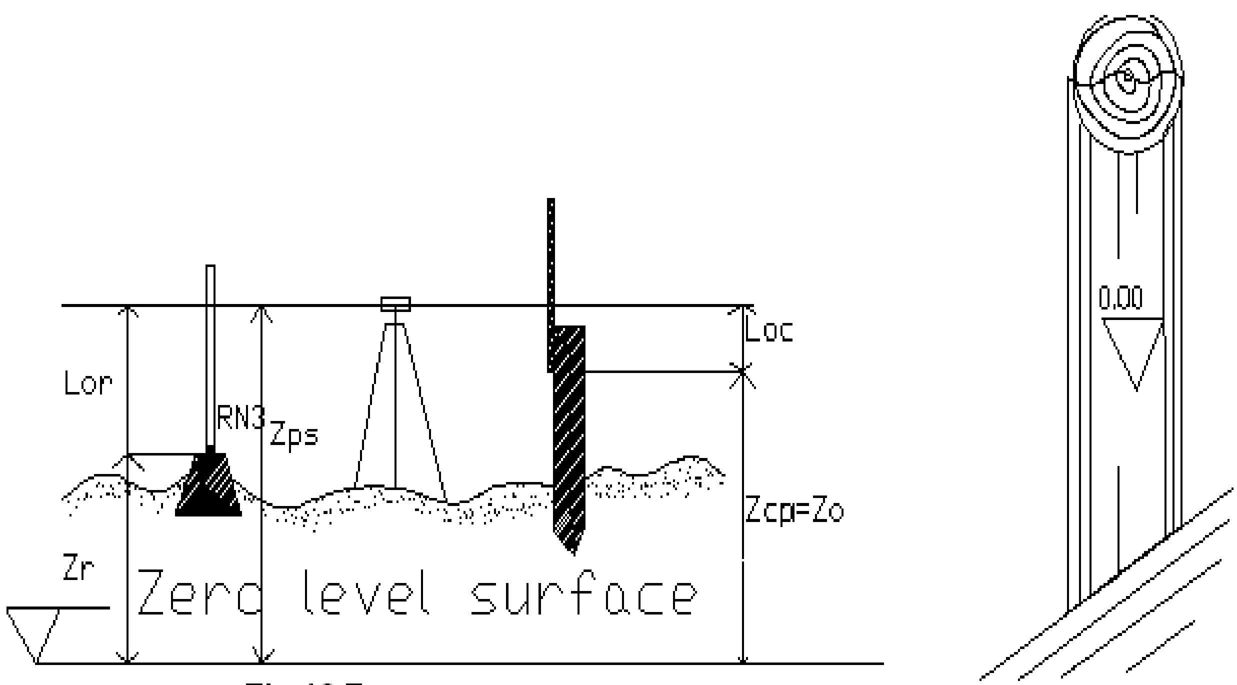


Fig 10.7

The method of the plane of sight consist in determining the on the vertical rod in the point that has already been sett out plan when the end of the rod is positioned on the zero elevation.

The setting out operation of the costruction zero elevation is made as follows: (fig 10.7)

-a strong peg is driven into ground in point c which is 4...6...1 away from the location of the building ,its peak remaining 1.00....1.50 above the ground.

-the level is sett up approximately in the middle of the distance between the bench mark RN3 of the support network and point in which peg has been installed.The line of sight is made perfectly calculate the elevation of the plane of sight:

Zps=Zr+LoR=62.935+1.535=64.470m

-the reading that has been made on the rod position in point c is calculated as bellow(when the bottom of the rod is held at the zero elevation)

LoC=Zps-ZcP=Zps-Z0=64.470-63.460=1.010m=1010mm

-a worker will move the rod on vertical along the peg until the operator will read on the rod at the horizontal cross hair the calculated value LoC=1010mm,the latter corresponding to the zero elevation.

The operated goes in point C where he inscribes a horizontal lines under the construction elevation zero +/-0.000=63.460m.

From point C the elevation for other points are transmitted for different stages of the construction.